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Research Article

1D electromagnetic generalized thermoelasticity problem for circular cylindrical hole under a rotating effect

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ABSTRACT

In this work, the generalized thermoelastic theory was used with one relaxation time in the context of (L. S) theory, to investigate the magneto-thermoelastic problem of a rotating infinite body with a circular cylindrical hole in the existence of a uniform magnetic field in the direction of the axis. Constant heat flux is flowing into the infinite body from a circular cylindrical hole. The governing equations compatible with magnetism and generalized thermoelasticity have been formulated. By using the Laplace transform and the scalar Laplace inversion, the governing equations have been solved. Numerical calculations were performed for the studied variables and the obtained results were presented graphically. The effect of rotation on temperature is very small, while it is very noticeable in the other functions. This study may be important in the study of pressure vessels and pipes in nuclear reactors, and chemical plants.

1. Introduction

The theory of thermoelasticity deals with the effect of thermal and mechanical disturbances on an elastic body. The interest in it in previous years has led to the emergence of a large number of research papers, both theoretical and experimental. The importance of Thermoelasticity is due to its many applications in various fields such as aviation, nuclear reactors, modern propulsion system technology, plasma physics, and geophysics. Lord and Shulman [1] introduced the theory of generalized thermoelasticity which is often referred to as the theory of generalized thermoelasticity with one relaxation time. Many papers have been concerned with the fundamental considerations of that theory Sherief and Khader [2] have obtained the solution to the propagation of discontinuities in electromagnetic generalized thermoelasticity in cylindrical regions, and they are used Boley [3] theorem to determine wavefront and speed in Laplace transform expressions. Furukawa and others [4] have studied the infinite body with a circular cylindrical hole. Zenkour and Abbas [5] discussed the generalized thermoelasticity problem of an annular cylinder with temperature-dependent density and material properties. Other works on the subject are [6-17]. The study of the effect of the magnetic field is of great interest in technical fields

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Nomenclature

J	electric current density	θ_0	constant
$\varepsilon_{0,}$	electric permeability	$H(t)$	Heaviside unit step function
μ_0	magnetic permeability	K_0 ,I_0	modified Bessel function
B	magnetic induction vectors	h^0	induced magnetic in the free space
D	electric induction vectors	E^0	induced electric fields
σ_0	electric conductivity	τ_0	relaxation time
u	displacement vector	F	Lorentz force
λ, μ	Lamé's modulii	k	thermal conductivity
T	absolute temperature	c_E	specific heat at constant strain
γ	material constant	ρ	density
T_0	reference temperature		

because of its applications in industrial technology. These applications include cooling of nuclear reactors, flow control of liquid metals and high-temperature plasmas, power generators, drying and solidification of binary alloys, and biological transfers. Ezzat and El-Bary [18] studied the functionally graded magneto-thermoelastic half-space with memory-dependent derivatives heat transfer. Biswas et al. [19] discussed the effect of rotation in a magneto-thermoelastic transversely isotropic hollow cylinder with the three-phase-lag model. Othman and Abbas [20] used the finite element method to solve the effect of rotation on a magneto-thermoelastic hollow cylinder with energy dissipation. Abd-Alla and Mahmoud [21] discussed a magneto-thermoelastic problem in rotating non-homogeneous orthotropic hollow cylinders under the hyperbolic heat conduction model. Said [22] discussed the deformation of a rotating two-temperature generalized magneto-thermoelastic medium with an internal heat source due to hydrostatic initial stress. Kumar [23] studied the effect of rotation in the magneto-micropolar thermoelastic medium due to mechanical and thermal sources. Abo-Dahab and Singh [24] discussed the Influences of magnetic field on wave propagation in generalized thermo elastic solid with diffusion. Khader [25] used different theories of magneto thermo elasticity for a uniform Laser Pulse for a Solid Cylinder. Kumar and Sharma [26] have studied thermo mechanical interactions in transversely isotropic magneto thermo elastic with and without energy dissipation with combined effects of rotation, vacuum, and two temperatures. Many authors contributed to this subject [27-42].

Formulation of the Problem

We consider the one-dimensional generalized thermoelasticity of an infinite body with a circular cylindrical hole, whose radius is as shown in figure (1). The boundary condition is that a constant heat flux flows into the infinite body from the hole, but the displacement at the hole is constrained. A constant magnetic field of strength H_0 acts in the direction of the z-axis. This produces an induced magnetic field h and an induced electric field E. Let (r, φ, z) be cylindrical polar coordinates with the z-axis coinciding with the axis of cylindrical hole.

Maxwell's equations

$$\begin{cases} \text{curl}\,\mathbf{h} = \mathbf{J} + \varepsilon_0 \dfrac{\partial \mathbf{E}}{\partial t}, \\ \text{curl}\,\mathbf{E} = -\mu_0 \dfrac{\partial \mathbf{h}}{\partial t}, \\ \text{div}\,\mathbf{h} = 0\,,\ \text{div}\,\mathbf{E} = 0. \end{cases} \quad (1)$$

Ohm's law for moving media states that

$$\mathbf{J} = \sigma_0 \left(\mathbf{E} + \mu_0 \frac{\partial \mathbf{u}}{\partial \mathrm{t}} \times \left(\mathbf{H}_0 + \mathbf{h} \right) \right) .$$

This equation can be linearized by neglecting small quantities of the second order giving

$$\mathbf{J}=\sigma_0\left(\mathbf{E}+\mu_0\frac{\partial\mathbf{u}}{\partial t}\times\mathbf{H}_0\right). \tag{2}$$

The displacement u and applied magnetic field H_0 have the components

$$u_r=u(r,t),\ u_\phi=u_z=0,$$
$$H_0=(0,0,H_0).$$

The induced magnetic and electric field and electric current has the components

$\mathbf{h}=(0,0,h)$, $\mathbf{E}=(0,E,0)$, $\mathbf{J}=(0,J,0)$.

The strain components

$$\begin{cases} e=\dfrac{\partial u}{\partial r}+\dfrac{u}{r}=\dfrac{1}{r}\dfrac{\partial(ru)}{\partial r}, \\ e_{rr}=\dfrac{\partial u}{\partial r},\ e_{\varphi\varphi}=\dfrac{u}{r}, \\ e_{zz}=e_{rz}=e_{z\varphi}=e_{r\varphi}=0. \end{cases} \tag{3}$$

The stress tensor σ_{ij} are given by

$$\sigma_{rr}=2\mu\frac{\partial u}{\partial r}+\lambda e-\gamma(T-T_0), \tag{4}$$

$$\sigma_{\varphi\varphi}=2\mu\frac{u}{r}+\lambda e-\gamma(T-T_0)\ , \tag{5}$$

$$\sigma_{zz}=\lambda e-\gamma(T-T_0)\quad , \tag{6}$$

$$\sigma_{r\varphi}=\sigma_{rz}=\sigma_{\varphi z}=0. \tag{7}$$

Equations of motion in the presence of magnetic field and rotation [24]

$$\mu\nabla^2\mathbf{u}+(\lambda+\mu)\,\mathrm{grad}\,\mathrm{div}\,\mathbf{u}+\mathbf{F}-\gamma\,\mathrm{grad}\,T =\rho\left(\ddot{\mathbf{u}}+\Omega\times(\Omega\times\mathbf{u})+2\Omega\times\dot{\mathbf{u}}\right) .\tag{8}$$

$\Omega\times(\Omega\times\mathbf{u})$ is centripetal acceleration due to time-varying motion only, $2\Omega\times\dot{\mathbf{u}}$ is Coriolis acceleration and F is the Lorentz force given by [2]

$$\mathbf{F}=\mathbf{J}\times\mathbf{B}. \tag{9}$$

Equation of heat conduction

$$k\nabla^2T=\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)\left(\rho c_E T+\gamma T_0 e\right) .\tag{10}$$

Initial conditions can be written as:

$$\begin{cases} u(r,0)=\dot{u}(r,0)=T(r,0)=\dot{T}(r,0)=0, \\ E(r,0)=\dot{E}(r,0)=h(r,0)=\dot{h}(r,0)=0. \end{cases}$$

Boundary conditions can be written as:

$$\begin{cases} -\dfrac{\partial T(r,t)}{\partial r}=\theta_0 H(t), \\ u\ (r,t)=0, \qquad at\, r=a. \\ h=h^0\ ,\ E=E^0, \end{cases}$$

(11)

2. Solution of the Problem

Equations (1) and (5) can be reduced as

$$\frac{\partial h}{\partial r}=-\left[J+\varepsilon_0\frac{\partial E}{\partial t}\right]\ , \tag{12}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rE)=-\mu_0\frac{\partial h}{\partial t}\ , \tag{13}$$

$$J=\sigma_o\left(E-\mu_0 H_0\frac{\partial u}{\partial t}\right). \tag{14}$$

From equations (13) and (14), by eliminate J, we obtain

$$\frac{\partial h}{\partial r}=\sigma_0\mu_0H_0\frac{\partial u}{\partial t}-\left(\sigma_0E+\varepsilon_0\frac{\partial E}{\partial t}\right). \tag{15}$$

From equations (13) and (15), by eliminat E , we obtain

$$\left(\nabla^2-\sigma_0\mu_0\frac{\partial}{\partial t}-\mu_0\varepsilon_0\frac{\partial^2}{\partial t^2}\right)h =\sigma_0\mu_0H_0\frac{\partial e}{\partial t}. \tag{16}$$

From equations (1) and (9), we obtain

$$F_r=J\mu_0(H_0+h),\quad F_\varphi=F_z=0. \tag{17}$$

Take div for both sides of equation (8), we get

$$\left[\mu_0^2\varepsilon_0H_0\frac{\partial^2}{\partial t^2}-\mu_0H_0\nabla^2\right]h-\gamma\nabla^2T+ (\lambda+2\mu)\nabla^2e=\rho\left[\frac{\partial^2e}{\partial t^2}-\Omega^2e-2\Omega\frac{\partial e}{\partial t}\right]. \tag{18}$$

Let us introduce the following non-dimension variables

$$(r,u)^* = c_0\eta\,(r,u),\ (\mathrm{t},\tau_0)^* = \mathrm{c}_0^2\eta(\mathrm{t},\tau_0),$$

$$\sigma^*_{\mathrm{ij}} = \frac{\sigma_{ij}}{(\lambda+2\mu)}\ ,\ \theta = \frac{\gamma(\mathrm{T}\text{-}\mathrm{T}_0)}{(\lambda+2\mu)},$$

$$\Omega^* = \frac{1}{c_0^2\eta}\Omega, \mathrm{E}^* = \frac{\eta}{\sigma_0\mu_0{}^2 H_0 c_0}E,$$

$$\mathrm{h}^* = \frac{\eta}{\sigma_0\mu_0 H_0}h\ ,\ \eta = \frac{\rho c_E}{k},\ c_0^2 = \frac{\lambda+2\mu}{\rho}.$$

The governing equations (4-6), (10), (13), (16) and (18) in non-dimensional form become:

$$\sigma_{rr} = 2\frac{\partial u}{\partial r} + (\beta^2-2)\,e - \beta^2\theta\ , \tag{19}$$

$$\sigma_{\varphi\varphi} = 2\frac{u}{r} + (\beta^2-2)\,e - \beta^2\theta\ , \tag{20}$$

$$\sigma_{zz} = (\beta^2-2)e - \beta^2\theta\ , \tag{21}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rE) = -\frac{\partial h}{\partial t}\ , \tag{22}$$

$$\left[\nabla^2 - \upsilon\frac{\partial}{\partial t} - V^2\frac{\partial^2}{\partial t^2}\right]h = \frac{\partial e}{\partial t}\ , \tag{23}$$

$$\nabla^2 e - \varepsilon_2\upsilon\left[\nabla^2 - V^2\frac{\partial^2}{\partial t^2}\right]h - \nabla^2\theta = \left(\frac{\partial^2}{\partial t^2} - \Omega^2 - 2\Omega\frac{\partial}{\partial t}\right)e\ , \tag{24}$$

$$\nabla^2\theta = \left(1+\tau_0\frac{\partial}{\partial t}\right)\left[\frac{\partial\theta}{\partial t} + \varepsilon_1\frac{\partial e}{\partial t}\right]\ . \tag{25}$$

$$\upsilon = \frac{\sigma_0\mu_0}{\eta},\ \varepsilon_1 = \frac{T_0\gamma^2}{c_E\rho^2 c_0^2},\ \varepsilon_2 = \frac{\mu_0 H_0^2}{\lambda+2\mu}\ ,V = \frac{c_0}{c},$$

$$\mathrm{c}^2 = \frac{1}{\varepsilon_0\mu_0}\ ,\ \beta^2 = \frac{(\lambda+2\mu)}{\mu}$$

Laplace transform with parameter s defined by the relation

$$\bar{f}(s) = \int_0^\infty f(t)\,e^{-st}\,dt$$

to both sides of equations (19)-(25), we get:

$$\bar{\sigma}_{rr} = 2\frac{\partial\bar{u}}{\partial r} + (\beta^2-2)\bar{e} - \beta^2\bar{\theta}\ , \tag{26}$$

$$\bar{\sigma}_{\varphi\varphi} = 2\frac{\bar{u}}{r} + (\beta^2-2)\bar{e} - \beta^2\bar{\theta}\ , \tag{27}$$

$$\bar{\sigma}_{zz} = (\beta^2-2)\bar{e} - \beta^2\bar{\theta}\ , \tag{28}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r\bar{E}) = -s\bar{h}\ , \tag{29}$$

$$\left[\nabla^2 - \upsilon s - V^2 s^2\right]\bar{h} = s\,\bar{e}\ , \tag{30}$$

$$\nabla^2\bar{e} + \varepsilon_2\upsilon\left[V^2 s^2 - \nabla^2\right]\bar{h} - \nabla^2\bar{\theta} = \left(s^2 - \Omega^2 - 2s\Omega\right)\bar{e}\ , \tag{31}$$

$$\nabla^2\bar{\theta} = \left(s+\tau_0 s^2\right)\left[\bar{\theta} + \varepsilon_1\bar{e}\right]. \tag{32}$$

from equations (30), (31), and (32), Eliminat $\bar{\theta}$, $\bar{\mathrm{h}}$ we obtain

$$(\nabla^6 - a\nabla^4 + b\nabla^2 - c)\bar{e} = 0\ , \tag{33}$$

$$a = s\left(\upsilon + sV^2\right) + s^2 - 2s\Omega - \Omega^2 + s\left(1+\varepsilon_1\right)\left(1+\tau_0 s\right) + s\upsilon\varepsilon_2\ ,$$

$$b = s\left(\upsilon+sV^2\right)\left[s^2 - 2s\Omega - \Omega^2 + s\left(1+\varepsilon_1\right)\left(1+\tau_0 s\right)\right] + s\left(1+\tau_0 s\right)\left(s^2-2s\Omega-\Omega^2\right) + s\upsilon\varepsilon_2\left[s^2V^2 + s\left(1+\tau_0 s\right)\right]$$

$$c = s^2\left(\upsilon+sV^2\right)\left(1+\tau_0 s\right)\left(s^2-2s\Omega-\Omega^2\right) + s^4\upsilon\varepsilon_2 V^2(1+\tau_0 s)\ .$$

In a similar manner we can show that $\bar{\theta}$, $\bar{\mathrm{h}}$ satisfy the equations

$$(\nabla^6 - a\nabla^4 + b\nabla^2 - c)\bar{h} = 0\ , \tag{34}$$

$$(\nabla^6 - a\nabla^4 + b\nabla^2 - c)\bar{\theta} = 0\ . \tag{35}$$

The solutions of equations (33)-(35), have the forms:

$$\bar{e} = \sum_{i=1}^{3} A_i K_0(k_i r)\ , \tag{36}$$

$$\bar{\theta} = \sum_{i=1}^{3}\frac{\varepsilon_1(s+\tau_0 s^2)}{k_i^2 - (s+\tau_0 s^2)}A_i K_0(k_i r)\ , \tag{37}$$

$$\bar{h} = \sum_{i=1}^{3}\frac{s}{k_i^2 - s(\upsilon+sV^2)}A_i K_0(k_i r)\ . \tag{38}$$

Where k_1^2, k_2^2 and k_3^2 can be obtain from the Eq.

$$k^6 - ak^4 + bk^2 - c = 0\ . \tag{39}$$

From equations (29) and (38), we obtain:

$$\overline{E} = \sum_{i=1}^{3} \frac{s^2}{k_i(k_i^2 - s(\upsilon + sV^2))} A_i K_1(k_i r) \quad (40)$$

From equations (3) and (36), we get:

$$\overline{u} = -\sum_{i=1}^{3} \frac{A_i}{k_i} K_1(k_i r) \quad (41)$$

From equations (36), (37), (41) and (26), we obtain:

$$\overline{\sigma}_{rr} = \sum_{i=1}^{3} A_i \left(\beta^2 \frac{k_i^2 - s(1+\varepsilon_1)(1+\tau_0 s)}{k_i^2 - (s+\tau_0 s^2)} K_0(k_i r) + \frac{2}{k_i r} K_1(k_i r) \right) \quad (42)$$

E^0 and h^0 in the free space inside the circular cylindrical hole satisfy the following equations

$$\frac{\partial \overline{h}^0}{\partial r} = -V^2 s \overline{E}^0 \quad , \quad (43)$$

$$\frac{1}{r} \frac{\partial}{\partial r} (r \overline{E}^0) = -s \overline{h}^0 \quad . \quad (44)$$

from equations (43) and (44), by Eliminat $\overline{E}^0$, we obtain:

$$\left[\nabla^2 - V^2 s^2 \right] \overline{h}^0 = 0 \quad . \quad (45)$$

Solution of Eq. (45) has given by

$$\overline{h}^0 = A_4(s) I_0(sVr) \quad . \quad (46)$$

From Eq. (42) and (39), we obtain

$$\overline{E}^0 = \frac{-A_4(s)}{V} I_1(sVr) \quad (47)$$

Eq. (11) after tak the Laplace transform, we get

$$\begin{cases} \dfrac{\partial \overline{\theta}(a,s)}{\partial r} = -\dfrac{\theta_0}{s}, \\ \overline{u}\,(a,s) = 0, \\ \overline{\mathrm{h}}\,(\mathrm{a,s}) = \overline{\mathrm{h}}^0(\mathrm{a,s}), \\ \overline{\mathrm{E}}(\mathrm{a,s}) = \overline{\mathrm{E}}^0(\mathrm{a,s}). \end{cases} \quad (48)$$

Applying the boundary conditions, we obtain the fsystem of linear equations in the unknown parameters A_1, A_2, A_3 ,and A_4

$$\sum_{i=1}^{3} \frac{k_i \varepsilon_1 (s + \tau_0 s^2)}{k_i^2 - (s + \tau_0 s^2)} A_i K_1(k_i a) = \frac{\theta_0}{s} \quad , \quad (49)$$

$$\sum_{i=1}^{3} \frac{A_i}{k_i} K_1(k_i a) = 0 \quad , \quad (50)$$

$$\sum_{i=1}^{3} \frac{s A_i K_0(k_i a)}{k_i^2 - s(\upsilon + sV^2)} = A_4 I_0(sVa), \quad (51)$$

$$\sum_{i=1}^{3} \frac{s^2 A_i K_1(k_i a)}{k_i(k_i^2 - s(\upsilon + sV^2))} = -\frac{A_4}{V} I_1(sVa). \quad (52)$$

Solving equations (49)-(52) to find A_1, A_2, A_3 ,and A_4

$$A_1 = \frac{\theta_0 \left[m_{34}(m_{43}m_{22} - m_{42}m_{23}) + m_{44}(m_{23}m_{32} - m_{22}m_{33}) \right]}{s\Delta}$$

$$A_2 = \frac{\theta_0 \left[m_{34}(m_{43}m_{23} - m_{43}m_{21}) + m_{44}(m_{21}m_{33} - m_{23}m_{31}) \right]}{s\Delta}$$

$$A_3 = \frac{\theta_0 \left[m_{34}(m_{21}m_{42} - m_{22}m_{41}) + m_{44}(m_{22}m_{31} - m_{21}m_{32}) \right]}{s\Delta}$$

$$A_4 = \frac{\theta_0}{s\Delta} \left[m_{41}(m_{22}m_{33} - m_{23}m_{32}) + m_{42}(m_{23}m_{31} - m_{21}m_{33}) + m_{43}(m_{21}m_{32} - m_{22}m_{31}) \right]$$

$$m_{1i} = \frac{k_i \varepsilon_1 (s + \tau_0 s^2)}{k_i^2 - (s + \tau_0 s^2)} K_1(k_i a), \quad i = 1,\ 2,\ 3$$

$$m_{2i} = \frac{K_1(k_i a)}{k_i}, \quad i = 1,\ 2,\ 3$$

$$m_{3i} = \frac{s K_0(k_i a)}{k_i^2 - s(\upsilon + sV^2)} \quad , \quad i = 1,2,3$$

$$m_{34} = I_0(sVa)$$

$$m_{4i} = \frac{s^2}{k_i(k_i^2 - s(\upsilon + sV^2))} K_1(k_i a) \ , \ i = 1,2,3$$

$$m_{44} = \frac{1}{V} I_1(sVa)$$

$$\Delta = m_{34} \left[m_{41}(m_{12}m_{23} - m_{13}m_{22}) + m_{42}(m_{13}m_{21} - m_{11}m_{23}) + m_{43}(m_{11}m_{22} - m_{31}m_{22}) \right] - m_{44} \left[m_{11}(m_{22}m_{33} - m_{23}m_{32}) + m_{12}(m_{13}m_{23} - m_{21}m_{33}) + m_{13}(m_{21}m_{32} - m_{31}m_{22}) \right]$$

3. Continuity and Discussion of Wave Propagation

To study the Continuity and Discussion of Wave Propagation Boley and Hetnarski [3, 43] will used. We found that the temperature has three finite discontinuities with respect to r on the wave fronts $r = r_1, r_2, r_3$ In these case

$r_1 = \frac{t}{\beta_{10}} + a$, $r_2 = \frac{t}{\beta_{20}} + a$, and $r_3 = \frac{t}{\beta_{30}} + a$

The jumps at these points have magnitudes [$\theta_{10}\, e^{\beta_{11}(a-r)}$], [$\theta_{20}\, e^{\beta_{21}(a-r)}$], and [$\theta_{30}\, e^{\beta_{31}(a-r)}$]. In the same procedure for the other functions, we see that u, h, and E are continuous functions for all values of and since for all of these functions [2].

It should be noted that the first derivatives of these functions have finite discontinuities at the locations r_i. Discontinuity in displacement means that one part of the substance penetrates into another part, and this contradicts physical phenomena. Finally, the stress σ, the same as θ, has three finite discontinuities [2].

4. Numerical results and discussion

The material properties are:

$$\lambda = 7.76\times10^{10}, \mu = 3.86\times10^{10}, \rho = 8954,$$
$$k = 386, \alpha_t = 1.78\times10^{-5}, C_E = 381,$$
$$\varepsilon_0 = 10^{-9}/36\pi, \sigma_0 = 5.7\times10^{7},$$
$$\mu_0 = 4\pi\times10^{-7}, T_0 = 293, \tau_0 = 0.02$$
$$\beta_{10} = 0.0549, \beta_{20} = 0.065724, \beta_{30} = 0.08401$$

To compute the values of the functions, a numerical procedure was used to invert the transforms in the above expressions. First, a numerical method based on Fourier expansion was used to invert the Laplace transforms [44].The FORTRAN programming language was used on a personal computer. The accuracy maintained was 5 digits for the numerical program. The problem was solved for two values of time namely for $t = 0.03$ and $t = 0.05$ with constant value of $\Omega = 10$. The graphs for the temperature, displacement, stress, induced electric field, and induced magnetic field are shown in figure (2) – figure (6), respectively. Dotted lines represent the solution for $t = 0.03$ and solid lines represent the case when $t = 0.05$.

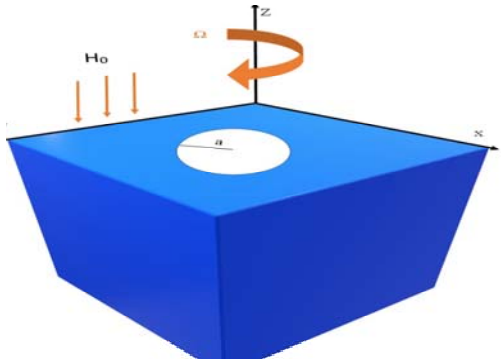

Fig. (1): Geometry of the problem

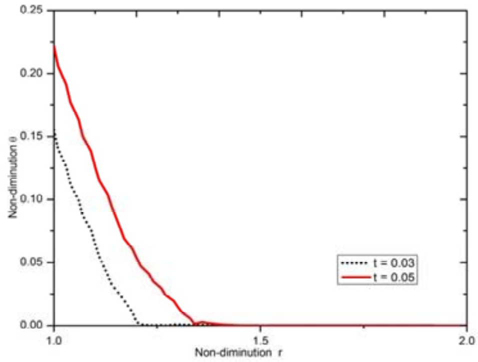

Fig. 2 Temperature Distribution

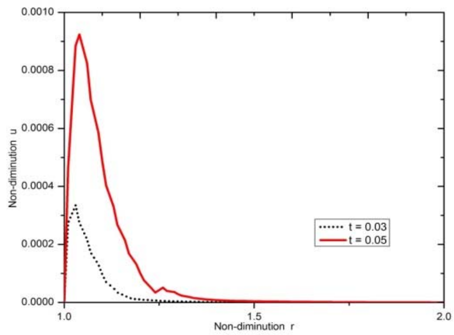

Fig. 3 Displacement Distribution

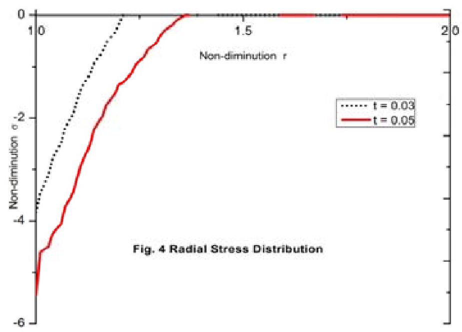

Fig. 4 Radial Stress Distribution

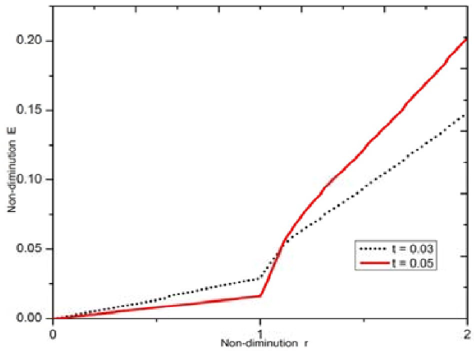

Fig. 5 Electric field distribution

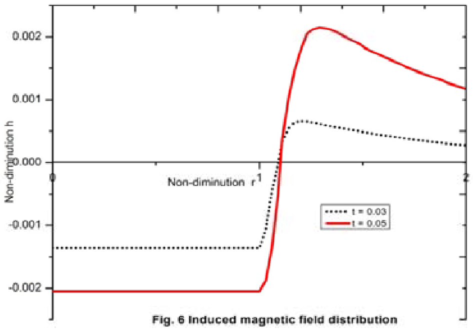

Fig. 6 Induced magnetic field distribution

In Figures (2, 4), we notice that the heat diffusion starts with a large value at the edge of the cylinder hole and gradually decreases inside the body until it vanishes. The effects of heating occupy a bounded region of space adjacent to the surface.

In Figure 3, we notice that the displacement is zero at the edge of the cylinder hole, and this applies to the boundary conditions, its value increases until it reaches its maximum value according to the value of time, and then gradually decreases until it vanishes.

In Figure 5, we notice that the value of the induced electric field is small inside the cylinder hole (vacuum), then gradually increases inside the body (sold), and then gradually decreases.

In Figure 6, we notice that the value of the magnetic field is a constant value inside the cylinder hole and then gradually decreases outside the cylinder.

We notice from the graphs that the solution fulfills the boundary conditions of the problem, as the functions have a value at the edge of the cylinder and cease as we move away from it. This is consistent with the research conducted in this field.

Figures (7-11) represent the temperature, displacement, stress, induced electric field and induced magnetic field, for different values of Ω , with a constant value of $t = 0.05$. Dotted lines represent the solution for $\Omega = 20$, dashed lines represent the solution for $\Omega = 10$, and solid lines represent the case when $\Omega = 0$.

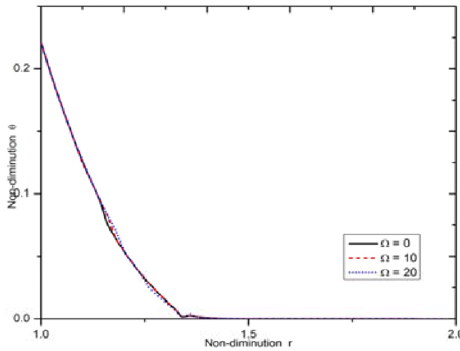

Fig. 7 Temperature Distribution

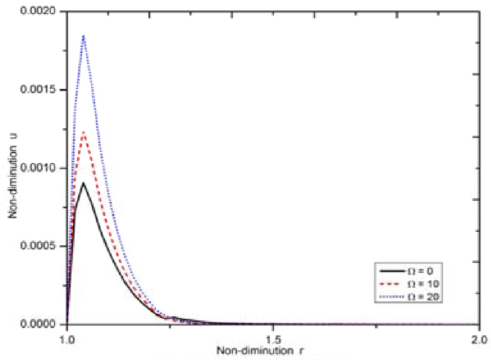

Fig. 8 Displacement Distribution

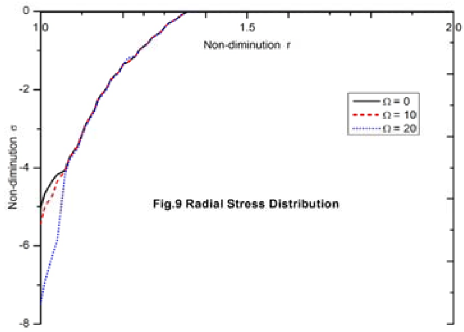

Fig.9 Radial Stress Distribution

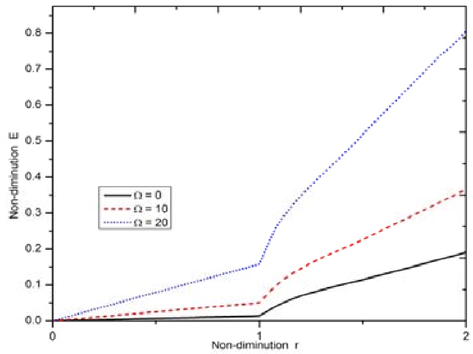

Fig.10 Electric field distribution

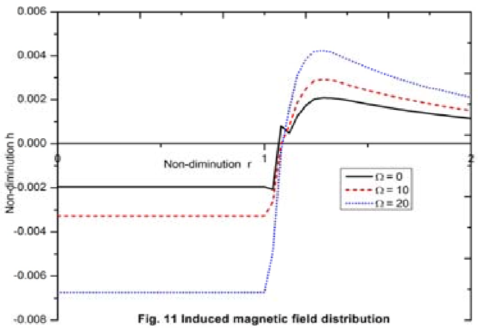

Fig. 11 Induced magnetic field distribution

In figure 7, we notice that the behavior of heat does not differ with the effect of rotation. This means that the rotation has very little effect on the temperature. In Figure 8, we notice that the rotation effect is large on the displacement. That is, an increase in the value of rotation leads to an increase in the value of displacement [9, 10]. In Figure 9, we notice that the effect of rotation is large around the edge of the cylindrical hole $r < 1.06$, then the effect of rotation gradually fades and the stress behavior is similar $r > 1.06$. In Figures 10 and 11, we notice that as the value of the rotation increases, the values of induced electric and magnetic fields increase.

5. Conclusions

The electromagnetic generalized thermoelasticity problem for a circular cylindrical hole under a rotating effect involves the study of the interaction between electromagnetic fields and thermal effects in a cylindrical structure with a hole subjected to rotation. This problem combines the principles of electromagnetism and thermoelasticity to analyze the behavior of the structure under the influence of both electromagnetic and thermal loads. We can obtain the following conclusions based on the above analysis. All quantities satisfied the boundary conditions. The values of the functions at the edge of the hole are large and gradually decrease inside the rigid body. The temperature and stress functions have three finite discontinuities. The displacement, induced electric field, and induced magnetic field are continuous functions. The effect of rotation on temperature is very small, while it is very noticeable on the displacement, stresses, induced magnetic and electric fields. The effect of rotation on the behavior of physical distributions is evident from the data and must be considered in manufacturing and design processes. This study is of importance in the study of structural components and mechanical elements such as pressure vessels and pipes in nuclear reactors, and chemical plants.

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