

Effect of Viscous Dissipation on the Onset of Jeffery Fluid Porous Convection in the Presence of Throughflow and Electric Field

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ABSTRACT

In this paper, A numerical analysis of Jeffrey fluid flow in a porous matrix with a collective impact of through flow, external electric field and viscous dissipation, was conducted to examine the onset of thermal convection. The critical Rayleigh number for stationary mode was determined using Galerkin processes and linear stability assumptions based on the normal mode procedure. The study investigated the effects of several key factors, including throughflow, viscous dissipation, electric field, and Jeffrey parameters, with their impacts analyzed through graphs. It was observed that higher values of the Jeffrey parameter, viscous dissipation term, and throughflow parameters tend to stabilize the system, while an increased electric field parameter accelerates the onset of convection.

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1. Introduction

In recent decades, the study of thermal convection in porous-walled channels has garnered significant attention from researchers due to its wide-ranging applications across various fields, including geosciences, bioengineering, food processing, petroleum engineering, desalination, solar power, cooling systems, and medical devices such as artificial kidneys and electronic cooling (Vafai [1,2], Straughan [3], Tan et al. [4], Ingham and Pop [5], Nikolay [6], Aneeta et al. [7], Sohail et al. [8,9]). The seminal work on the initiation of thermal convection in porous layers was conducted by Horton and Rogers [10], who determined a critical Darcy–Rayleigh number of $4\pi^2$ to initiate convective motion. Subsequently, this research area saw further exploration and

expansion by numerous scholars, including Lapwood [11], Nield [12,13], Shivakumara et al. [14], Gangadharaiah [15,16,17,18], Suma et al. [19], and Gangadharaiah and Ananda [20], under diverse physical conditions. The reviews by Nield and Bejan [21], Vadasz [22], and Straughan [23] may be consulted for more details.

Numerous research investigations have been conducted to evaluate the influence of electric fields on convective instability issues within porous media. This area of study holds great significance across multiple domains, including geosciences, energy transfer principles, plasma physics, and the modeling of porous substances (Probstein and Hicks [24], Gao et al. [25]).

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Nomenclature			
\vec{V}	velocity of the Jeffery fluid	λ	Jeffery parameter
μ	fluid viscosity of Jeffery's fluid	P	pressure
\vec{F}_E	electric force due to an electric field	ρ_0	reference density
β_Θ	heat expansion coefficient	τ	time
η	heat capacity fraction	α_e	effective thermal diffusivity
R_Θ	thermal Rayleigh–Darcy number,	R_E	electric Rayleigh–Darcy number
v_0	Throughflow vertical velocity	Ge	Gebhart number
Pe	throughflow parameter	R_ρ	density Rayleigh–Darcy number
d	Depth of porous matrix	ψ_1	fixed electrical potential
\hat{V}	Perturbed vertical velocity	\hat{T}	Perturbed temperature
γ	dielectric constant	K	permeability

The influence of the electric field becomes particularly prominent when the working fluid exhibits dielectric properties and possesses low electrical conductivity. In a study by Moreno et al. [26], they investigated the impact of an alternating current (AC) electric field on the flow of oil and water within a porous matrix. Their findings suggested that applying an electric field had the potential to enhance petroleum production. Rudraiah and Gayathri [27] studied the influence of temperature modulation on electro-convection in a densely packed porous matrix layer. Shivakumara et al. [28] investigated how rotary motion and electric fields jointly influenced thermal instability in a dielectric fluid within a Brinkman porous layer, while Yadav et al. [29] and Chand et al. [30] explored the consequences of electric fields on nanofluid convective behavior in porous media. Additionally, studies on convective instability in Jeffery fluid convection have been limited, with Martinez-Mardones and Perez-Garcia [31] examining such instability under various velocity boundary conditions. A recent study by Yadav [32] also explored the impact of electric fields on Jeffery fluid penetrative convection within the porous media layer.

Understanding the influence of throughflow on convective instability in porous layers is crucial for applications in engineering, geophysics, and electrohydrodynamics, including in-situ electronics processing, chemical equipment, energy assets, petroleum, geothermal energy, and addressing real-world challenges associated with porous medium

throughflow (Vafai [1]; Nield and Bejan [33], Gangadharaiah et al.[34,35], Gangadharaiah and Suma[36], Gangadharaiah and Nagarathnamma[37], Gangadharaiah [38,39,40,41], Nagarathnamma et al.[42], Shivakumara et al.[43] and Gangadharaiah et al.[44,45]).

Viscous dissipation plays a vital role in natural convection and stronger gravitational fields, converting mechanical energy into heat. Gebhart [46] has shown that the dissipation effect remains independent of Grashof and Prandtl numbers. In contrast, Roy and Murthy [47] have shown that viscous dissipation has a dual influence when coupled with the Soret effect, impacting flow fields significantly in a porous matrix. Understanding these interactions is crucial for predicting and controlling fluid flow and heat transfer in various scenarios.

Very recently, Srinivasacharya and Dipak [48] have studied the impact of gravity field with throughflow along with viscous dissipation on the onset of thermosolutal convection in a porous medium layer. Yadav [49] studied the the impact of throughflow and electric fields on nanofluid fluid in porous media. The study of the thermal instability in a horizontal porous layer saturated with a non-Darcian effects are reviewed in (Ali J. Chamkha[50,51,52] Khalil et al.[53], Prathap Kumar et al.[54], Keimanesh et al.[55], Umavathi et al.[56]). Some of the The study of the thermal instability in a horizontal porous layer saturated with a Jeffery's fluid are reviewed in (Devi et al.[57], Rana and Poonam

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Kumari [58], Sharma et al.[59], Gautam et al.[60] and Promila Devi et al.[61]).

In areas like chemical engineering, electric machinery, oil extraction, and crystal development where convection has to be regulated, the examination of the impact of electric field, viscous dissipation, and throughflow on Jeffrey fluid convection in porous media seems to be crucial. The purpose of this study is to identify the key elements of the combined effect of viscous throughflow and electric field on the initiation of Jeffrey fluid convection in porous media. The key settings for the onset of convective motion are generated and described with the aid of figures and tables using linear stability theory.

2. Mathematical Formulation

The convective motion of a dielectric Jeffrey fluid in an infinitely large porous medium layer heated from below with throughflow vertical velocity v_0 . The layer is between two boundaries $y = 0$ and $y = d$ and the temperatures at the bottom and top boundaries are supposed to be Θ_L and Θ_U , respectively. The configuration described involves a Jeffrey fluid layer exposed to an externally applied, uniform, vertically oriented AC electric field. Specifically, the lower edge of the layer is connected to the ground, while the upper edge is maintained at a fixed electrical potential ψ_1 . The arrangement is illustrated in detail in Fig. 1.

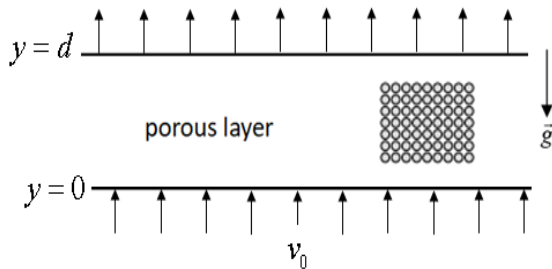


Fig. 1. Physical configuration

The flow governing equations under this model are [25,26,49,58]

$$\nabla \cdot \vec{V} = 0 \quad (1)$$

$$\frac{\mu}{(1+\lambda)K} \vec{V} = -\nabla P + \rho_0 [1 - \beta_\Theta (\Theta - \Theta_0)] \vec{g} + \vec{F}_E, \quad (2)$$

$$\eta \frac{\partial \Theta}{\partial t} + (\vec{V} \cdot \nabla) \Theta = \alpha_e \nabla^2 \Theta + \frac{\mu}{K} \vec{V} \cdot \vec{V}, \quad (3)$$

According to Landau et al [33], the electric force \vec{F}_E is taken as

$$\vec{F}_E = \frac{1}{2} \nabla \left[\rho \frac{\partial \gamma}{\partial \rho} (\vec{E}_f \cdot \vec{E}_f) \right] - \frac{1}{2} (\vec{E}_f \cdot \vec{E}_f) \nabla \gamma. \quad (4)$$

For zero free charge density, the associated Maxwell equations are

$$\nabla \times \vec{E}_f = 0, \quad (5)$$

$$\nabla \cdot (\gamma \vec{E}_f) = 0, \quad (6)$$

$$\vec{E}_f = -\nabla \psi, \quad (7)$$

where

$$\gamma = \gamma_0 [1 - e(\Theta - \Theta_0)], \quad (8)$$

$$\Theta = \Theta_L \quad \text{at } y = 0, \quad \Theta = \Theta_U \quad \text{at } y = d.$$

and

$$v = 0, \quad \text{at } y = 0, d. \quad (9)$$

Now, we consider the following non-dimensional variables:

$$\bar{x} = \frac{x}{d}, \quad \bar{\tau} = \frac{\tau \alpha_e}{\eta d^2}, \quad \bar{V} = \frac{v d}{\alpha_e}, \quad \bar{P} = \frac{PK}{\mu \alpha_e}, \quad \bar{E}_f = \frac{\vec{E}_f}{e \Delta \Theta E_{f0}},$$

$$\bar{\Theta} = \frac{\Theta - \Theta_0}{\Delta \Theta}, \quad \bar{\psi} = \frac{\psi}{e \Delta \Theta E_{f0} d}, \quad \bar{\gamma} = \frac{\gamma}{\gamma_0}, \quad (10)$$

Using these, the dimensionless forms of eqs (1)–(9) become

$$\bar{\nabla} \cdot \bar{\vec{V}} = 0 \quad (11)$$

$$0 = -\bar{\nabla} \left[\bar{P} + y R_\rho \hat{e}_y - \frac{1}{2} R_E \rho \frac{\partial \bar{\gamma}}{\partial \rho} (\bar{\vec{E}}_f \cdot \bar{\vec{E}}_f) \right] - \quad (12)$$

$$\frac{\bar{\vec{V}}}{(1+\lambda)} + R_\Theta \Theta \hat{e}_y - \frac{1}{2} R_E (\bar{\vec{E}}_f \cdot \bar{\vec{E}}_f) \bar{\nabla} \bar{\gamma},$$

$$\eta \frac{\partial \bar{\Theta}}{\partial t} + (\bar{\vec{V}} \cdot \bar{\nabla}) \bar{\Theta} = \alpha_e \bar{\nabla}^2 \bar{\Theta} + \frac{\mu}{K} \bar{\vec{V}} \cdot \bar{\vec{V}}, \quad (13)$$

$$\bar{\nabla} \times \bar{\vec{E}}_f = 0, \quad (14)$$

$$\bar{\nabla} \cdot (\bar{\gamma} \bar{\vec{E}}_f) = 0, \quad (15)$$

$$\bar{\vec{E}}_f = -\bar{\nabla} \bar{\psi}, \quad (16)$$

$$\bar{\gamma} = [1 - \Delta \Theta_e \bar{\Theta}], \quad (17)$$

$$\bar{\Theta} = 1 \quad \text{at } \bar{y} = 0, \quad \bar{\Theta} = 0 \quad \text{at } \bar{y} = 1.$$

and

$$\bar{V}' = 0, \text{ at } y = 0, 1. \quad (18)$$

where

$R_\Theta = \frac{\rho_0 g \beta_\Theta \Delta \Theta K d}{\mu \alpha_e}$ is the thermal Rayleigh–Darcy number,

$R_E = \frac{\gamma_0 K (e \Delta \Theta E_{f0})^2}{\mu \alpha_e}$ is the AC electric Rayleigh–Darcy number,

$Ge = \frac{\beta_\Theta g d}{\gamma}$ is the Gebhart number,

$R_\rho = \frac{\rho_0 g K d}{\mu \alpha_e}$ is the density Rayleigh–Darcy

number, $Pe = \frac{v_0 d}{\tau \alpha_e}$ is the throughflow parameter.

In this basic state, it is assumed that the Jeffery fluid is at rest and heat is transported only by conduction. Thus, the basic state of Jeffery fluid is expressed by

$$\begin{aligned} \bar{V}_b = 0, \bar{\Theta}_b = \bar{\Theta}_b(\bar{y}), \bar{P}_b = \bar{P}_b(\bar{y}), \\ \bar{\gamma}_b = \bar{\gamma}_b(\bar{y}), \bar{E}_{f,b} = \bar{E}_{f,b}(\bar{y}) \hat{e}_y, \bar{\psi}_b = \bar{\psi}_b(\bar{y}). \end{aligned} \quad (19)$$

On solving, we get

$$\begin{aligned} \Theta_b = \frac{1}{(e^{Pe} - 1)} \left(e^{Pe} + \frac{PeGe}{R_\Theta} \right) - \\ \frac{1}{(e^{Pe} - 1)} \left(1 + \frac{PeGe}{R_\Theta} \right) e^{Pe y} + \frac{PeGe}{R_\Theta} y \end{aligned} \quad (20)$$

We now take tiny perturbations on the basic flow as

$$\begin{aligned} \bar{V} = \bar{V}' + v_0, \bar{\Theta} = \bar{\Theta}_b + \bar{\Theta}', \bar{P} = \bar{P}_b + \bar{P}' \\ , \bar{\gamma} = \bar{\gamma}_b + \bar{\gamma}', \bar{E}_f = \bar{E}_{f,b} + \bar{E}_f', \bar{\psi} = \bar{\psi}_b + \bar{\psi}'. \end{aligned} \quad (21)$$

On applications of these, we achieve the subsequent linear stability equations

$$\frac{\bar{\nabla}^2 \bar{v}'}{(1 + \lambda)} = R_\Theta \bar{\nabla}_H^2 \bar{\Theta}' + R_E \bar{\nabla}_H^2 \left(\bar{\Theta}' - \frac{\partial \bar{\psi}'}{\partial \bar{y}} \right), \quad (22)$$

$$\frac{\partial \bar{\Theta}'}{\partial \bar{t}} + (\bar{v}' \cdot \nabla) \bar{\Theta}' + (v_0 \cdot \nabla) \bar{\Theta}_b = \bar{\nabla}^2 \bar{\Theta}' + \frac{Ge}{R_\Theta} \bar{v}' \cdot \bar{v}' \quad (23)$$

$$\bar{\nabla}^2 \bar{\psi}' - \frac{\partial \bar{\Theta}'}{\partial \bar{y}} = 0, \quad (24)$$

The equation of the normal mode form is given by

$$\begin{aligned} (\bar{v}', \bar{\Theta}', \bar{\psi}') \\ = \{ \hat{V}(y), \hat{T}(y), \hat{\psi}(y) \} \exp[i(\bar{k}x + m\bar{y}) + i\sigma t], \end{aligned} \quad (25)$$

On application of Eq. (25) into Eqs. (22) – (24), we can write:

$$\frac{(D^2 - a^2) \hat{V}}{(1 + \lambda)} + R_\Theta a^2 \hat{T} + R_E a^2 (\hat{T} - D\hat{\psi}) = 0, \quad (26)$$

$$(D^2 - a^2 - PeD) \hat{T} + \frac{2GePe}{R_\Theta} \hat{V} - \hat{V} f(y) = 0, \quad (27)$$

$$(D^2 - a^2) \hat{\psi} + D\hat{T} = 0 \quad (28)$$

The linearized boundary conditions are:

$$\hat{V} = \hat{T} = D\hat{\psi} = 0 \text{ at } z=0 \text{ and } z=1 \quad (29)$$

Where

$$f(y) = \frac{Pe}{(1 - e^{Pe})} \left(1 + \frac{PeGe}{R_\Theta} \right) e^{Pe y} + \frac{PeGe}{R_\Theta}.$$

3. Method of Solution

The Galerkin-type weighted residuals approach is employed to find a numerical solution, in which three variables, \hat{V} , \hat{T} and $\hat{\psi}$ are considered as

$$\begin{aligned} \hat{V} = \sum_{i=1}^N A_i \hat{V}_i, \quad \hat{T} = \sum_{i=1}^N B_i \hat{T}_i \quad \text{and} \\ \hat{\psi} = \sum_{i=1}^N C_i \hat{\psi}_i \quad n = 1, 2, 3, \dots \end{aligned} \quad (30)$$

where A_i, B_i and C_i are unknown constant.

We have applied the Galerkin approach, functions \hat{V} , \hat{T} and $\hat{\psi}$ are assumed as

$$\begin{aligned} \hat{V}_i = \sin(\pi i y), \quad \hat{T}_i = \sin(\pi i y), \\ \hat{\psi}_i = \cos(\pi i y) \end{aligned} \quad (31)$$

Substituting solution (30) into Eqs (26) - (28) and using (31), integrating each equation from 0 and 1, we get the matrix equations, On solving the obtained matrix equations, we get a

non-trivial solution, the critical Rayleigh number R is derived for stationary convection in terms of the governing parameters $(R_E, \lambda, Pe, Ge, a)$.

4. Results and Discussion

The effects of electric field and viscous dissipation on: the throughflow parameter, the viscous dissipation parameter, and the Jeffery parameter on stationary Jeffery fluid convection have been plotted graphically and their instabilities have been explored in this paper. Test calculations are performed for the scenario in which viscous dissipation is absent for various values of λ and compared with those of Yadav [49] in Table 1. to verify the correctness of the numerical approach utilized. See Table 1 for our findings, which show good agreement.

Fig. 2. illustrates the relationship between R^c and λ for different R_E values. Increasing R_E and λ causes R^c to decrease, expediting convection onset. This occurs because higher R_E intensifies electrostatic forces, while greater λ reduces Jeffery fluid retardation time, both promoting convection initiation.

The influence of the throughflow parameter on the beginning of convective growth in Jeffery fluid is displayed in Fig. 3. It is noticed that R^c enhances with an enhancement in the value of Pe . Accordingly, Pe has a stabilizing effect whereas R^c decreases with increasing the Jeffery parameter λ .

In Fig. 4, the impact of the viscous dissipation parameter Ge on the critical Rayleigh number's variation with the Jeffery parameter λ is evident. As Ge increases, the critical Rayleigh number also increases, while increasing λ results in a decrease in the critical Rayleigh number. This implies that the presence of a viscous dissipation parameter imparts a stabilizing character to the flow. This stabilization occurs because the work done by the fluid on adjacent layers is converted into heat due to shear forces, promoting convective motion advancement.

Fig. 5 presents the variation of R^c with Pe for various values of R_E . From Fig. 5, it can be seen that on increasing the values of R_E , the value of R^c decreases. Thus, an increase in R_E destabilizing effect but on increasing the values of Pe , the value of R^c increases, thus Pe has a stabilizing effect on configuration.

Figure 6 presents the variation of R^c with Pe in absence and presence of viscous deception, Jeffery parameter, and electrified. From Fig. 6, it can be seen that in the absence of viscous deception, the Jeffery parameter and electrified configuration have a more stabilizing effect and the stability curve has symmetry for both upward and downward throughflow, however, in the presence of these parameters system has stabilizing effects on only upward throughflow.

Table 1 Contrast of R^c and a_c for various values of λ with $Ge = 0$ and $R_E = 6$

λ	Yadav [49]		Present study	
	R^c	a_c	R^c	a_c
0	36.42	3.26	36.42	3.26
0.2	29.83	3.29	29.83	3.29
0.4	25.12	3.31	25.12	3.31
0.6	21.58	3.34	21.58	3.34
0.8	18.83	3.36	18.83	3.36
1.0	16.63	3.39	16.63	3.39

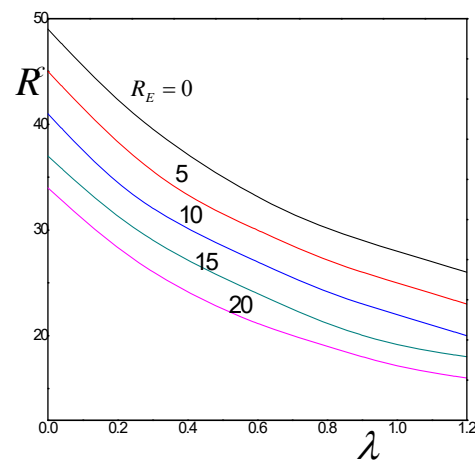


Fig 2. Variation of R^c verses λ for various values of R_E with $Pe = 3$ and $Ge = 1$.

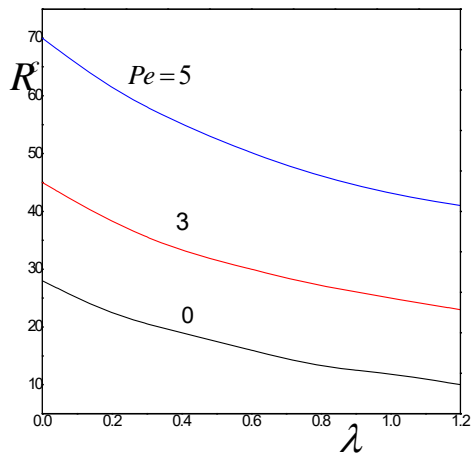


Fig 3. Variation of R^c versus λ for various values of Pe with $R_E = 5$ and $Ge = 1$.

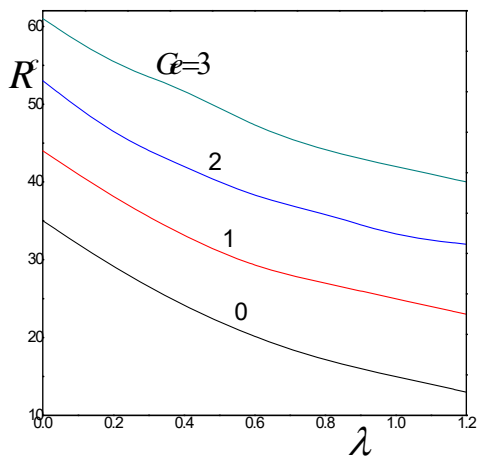


Fig 4. Variation of R^c versus λ for various values of Ge with $R_E = 5$ and $Pe = 3$.

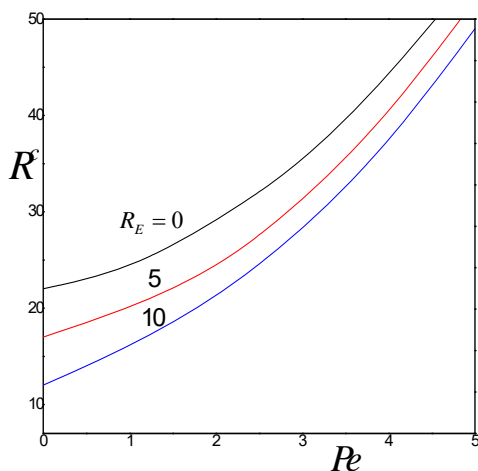


Fig 5. Variation of R^c versus Pe for various values of R_E with $Ge = 1$ and $\lambda = 0.5$.

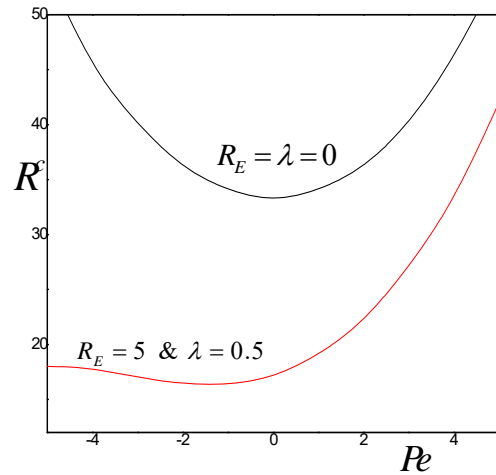


Fig 6. Variation of R^c versus Pe with and without viscous dissipation and electrified.

5. Conclusions

A numerical analysis of Jeffrey fluid flow in a porous medium, incorporating throughflow, an external electric field, and viscous dissipation, was conducted using the Galerkin technique. The study identified that higher throughflow parameters and increased viscous dissipation stabilize convection initiation, while a higher Jeffrey parameter accelerates it. Stronger electric fields destabilize the system. This research implies that effective control of convective instability can be achieved by adjusting the electric field intensity and Jeffrey parameter in conjunction with viscous dissipation and throughflow in the system setup.

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