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Research Article

Influence of variable viscosity and gravity fluctuation on double diffusive convection in a fluid layer with boundary slab of finite conductivity

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ABSTRACT

The linear stability analysis is carried out for the onset of double-diffusive convection in a fluid layer with a boundary slab along with temperature-dependent viscosity and gravity fluctuation. The authors proposed three types of gravity fluctuation. We considered three cases of gravity field fluctuation: (a) linear and (b)parabolic and (c) cubic. An analytical solution for the subsequent problem is acquired through the perturbation technique. The findings demonstrate that the viscosity variation parameter, the thermal conductivity ratio, the gravity parameter, the depth ratio, and the soret parameter accelerate the start of convection, while the increasing Lewis number slow down the convective motion. Additionally, the system was found to be more stable for the linear type of gravity field fluctuation and more unstable for the cubic type of gravity field fluctuation.

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1. Introduction

The investigation of thermosolutal convection in a fluid layer is a well-known topic in fluid dynamics with numerous applications, including biological systems and biotechnology Vafai [1], geological processes (Straughan [2] and Panfilov [3]), and engineering(Wu [4] and Xu et al.[5]). Understanding the start of convective motion with variable viscosity fluids is thus essential for comprehending planetary bodies' inner dynamic behavior and thermal development. Convective motion with temperature-dependent viscosity has technical aspects such as chemical and nuclear reactions (Baker et al.[6] and Delichatsios [7]), liquid-metal batteries (Kim et al.[8] and Nield and Kuznetsov [9]), nanofluids (Smith and Hammitt [10]), and fire and combustion modeling(Siddheshwar et al.[11]).

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Nomenclature

α_T	coefficient of thermal expansion	μ	fluid viscosity
μ	fluid viscosity	$\vec{V}$	velocity vector (u, v, w)
θ	amplitude of perturbed temperature	Le	Lewis number
k_C	solutal diffusivity	R	Rayleigh number
W	perturbed vertical velocity	p	pressure
D_{TC}	soret diffusivity	k	thermal diffusivity
δ	gravity parameter	T	temperature
ρ_0	fluid density	D	differential operator d/dz
Sr	Soret parameter	$G(z)$	variable gravity function
$\Pr$	Prandtl number	a	horizontal wave number
B	viscosity parameter	C	concentration
d_r	depth ratio	Rs	solute Rayleigh number
α_T	coefficient of thermal expansion	k_r	thermal conductivity ratio

The Investigation of such convection with changing viscosity fluids is very important since a large variety of industrial fluids, such as petroleum and ceramics, and nanofluids, have changing viscosities (Gangadharaiah and Ananda [12], Gangadharaiah et al.[13]). While we were aware that viscosity could vary due to other factors, in earlier research, fluids were sometimes assumed to have a constant viscosity. A fluid may have a viscosity that is temperature-dependent and drops off exponentially as the temperature rises (Griffiths [14]).

The heat transfer and spatial organization of fluid are influenced by temperature-dependent viscosity. Few scientists have looked into how temperature affects viscosity in different kinds of issues. The temperature-dependent viscosity effect in Benard instability and Marangoni instability were both studied by Torrance and Turcotte [15], Stengel et al. [16], Kozhoukharova and Rozé [17], and Lam and Bayazitoglu [18]. In a two-layered system with internal heat generation, the influence of temperature-dependent viscosity has been investigated by Gangadharaiah [19]. Shivakumara et al.[20] have examined the effects of non-uniform basic temperature gradients on surface-driven convection with boundary slab. Chaya and Gangadharaiah [21] have explored cross-diffusive terms in double-diffusive penetrative convection with changing viscosity. Only a few papers discuss how heat conductivity caused by plates affects the stability of configuration (Gangadharaiah([22]and [23]), Suma et al. [24], Ananda et al. [25], Rana et al.[26], Rana and Thakur[27 and 28] , Rana and Chand [29 and 30], and Gangadharaiah [31]).

Alex and Patil [32] examined the gravity fluctuation with internal heating of the configuration of the porous bed and discovered that a decrease in the gravity factor increases the stability of the configuration. Using the regular perturbation technique, Suma et al. [33] and Gangadharaiah et al. [34] reported the impact of linear gravity fluctuation with throughflow and internal heating in a porous bed configuration. The effects of a changing gravity field on porous layers were examined by Nagarathnamma et al. [35] using the Galerkin approach. The magnetic field and throughflow effects on porous bed configuration is studied by Yadav [36]. In a fluid layer, there has been very little research done on the effects of variable gravity. Mahajan and Tripathi [37] looked into how gravity fluctuation affected the stability of a thermosolutal convective flow in a situation where convective motion arises from non-uniformity in the thermophoresis parameter.

Very recently, the penetrative solutal convective motion with varying gravity and throughflow in a fluid layer was analyzed by Gangadharaiah et al. [38]. The study of temperature-dependent viscosity with varying gravity in the context of cross-diffusive terms is critical to understanding the convection process in engineering sciences (Gangadharaiah et al.([39] and [40]), Shivakumara et al.[41], Samart et al.[42], Gangadharaiah and Suma [43], Makinde [44], Makinde and Mhone [45 and 46]).

This paper aimed to study the temperature-dependent viscosity and boundary slab effects on thermosolutal convection in a horizontal fluid layer with gravity fluctuation. Such examinations may be very helpful to crack the problems associated with large-scale flows, such as material processing, Earth's crust, atmosphere, ocean, pollutant passage in saturated soils, fuel piercing, and crystals growing, where throughflow can be vital to manage the convective instability. We have considered three types of gravity fluctuation and the analytical solution for the eigenvalue problem is acquired through the regular perturbation technique.

2. Mathematical Formulation

We consider an infinitely extended fluid layer at $z=0$ and $z=d$, with a solid slab at the bottom having thickness d_s, the configuration is heated and salted from below as demonstrated in Fig1. We assume that the viscosity depends exponentially on the temperature of the form $\mu=\mu_0\exp\left[-A\left(T-T_0\right)\right]$, and the gravity vector $\vec{g}$ is, $\vec{g}=-g_0\left(1+\lambda H\left(z\right)\right)\hat{k}$, which spreads with the vertical reverse *z*-direction. The flow governing equations are given below(Char and Chen[47]).

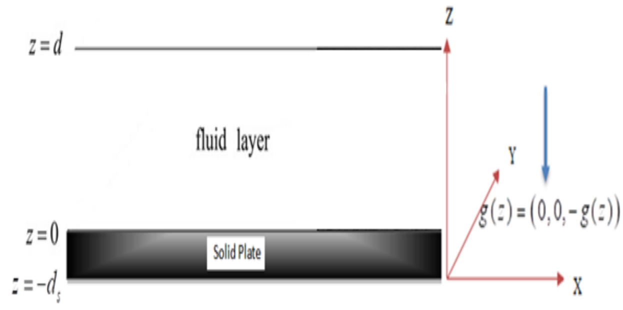

Fig. 1. Physical configuration

Fluid Layer $\left(0\le z\le d\right)$**:**

$$\nabla\cdot\vec{V}=0 \tag{1}$$

$$\rho_0\left(\frac{\partial\vec{V}}{\partial t}+\left(\vec{V}\cdot\nabla\right)\vec{V}\right)=-\nabla p+2\nabla.\left[\mu\left(\nabla.\vec{V}+\nabla.\vec{V}^T\right)\right]$$
$$+\rho_0\left\{1-\alpha_T\left(T-T_0\right)+\alpha_C\left(C-C_0\right)\right\}g(z)k, \tag{2}$$

$$\frac{\partial T}{\partial t}+\vec{V}.\nabla T=\kappa_T\nabla^2 T \tag{3}$$

$$\frac{\partial C}{\partial t}+\vec{V}.\nabla C=\kappa_C\nabla^2 C+D_{TC}\nabla^2 T \tag{4}$$

Solid layer $\left(-d_s\le z\le 0\ \right)$**:**

$$\frac{\partial T_s}{\partial t}=D_s\nabla^2 T_s\ . \tag{5}$$

where $\vec{V}=\left(u,v,w\right)$ is the velocity vector; T is the temperature; μ is the dynamic viscosity, κ_T is the thermal diffusivity, C is the solute concentration, k_C is the solutal diffusivity and D_{TC} is the Soret diffusivity.

The basic state of the fluid is

$$\left(u,v,w,T,p,C,\mu\right)=\left\{0,0,0,T_b\left(z\right),p_b\left(z\right),C_b\left(z\right),\mu_b\left(z\right)\right\} \tag{6}$$

$$\left(u,v,w,T,p,C,\mu\right)=\left\{0,0,0,T_b,p_b,C_b,\mu_b\right\}$$
$$+\left\{u',v',w',T',p',C',\mu'\right\} \tag{7}$$

On eliminating the pressure term and using the above equation, equations(1)-(4) can be written as

$$\frac{1}{pr}\frac{\partial}{\partial t}\nabla^2 w = \tilde{f}\,\nabla^4 w + 2\frac{\partial\tilde{f}}{\partial z}\nabla^2\frac{\partial w}{\partial z}$$

$$+\frac{\partial^2\tilde{f}}{\partial z^2}\left(\nabla^2 w - 2\nabla_h^2 w\right) + R\nabla_h^2 T + Rs\nabla_h^2 S \tag{8}$$

$$\frac{\partial T}{\partial t} = \nabla_h^2 T + w \tag{9}$$

$$\frac{\partial C}{\partial t} = Sr\,\nabla_h^2 T + Le\nabla_h^2 C + w \tag{10}$$

where $R = \dfrac{\alpha g\,\Delta T d^3}{\nu\,\kappa_T}$ is the Rayleigh number, $Rs = \dfrac{\rho g\,\Delta S d^3}{\nu\,\kappa_C}$ is the Solutal Rayleigh number $Le = \dfrac{\kappa_C}{\kappa_T}$ is the Lewis number, $Sr = \dfrac{D_{TS}\,\Delta S}{\kappa_C \Delta T}$ is the Soret number, $Pr = \dfrac{\nu}{\kappa_T}$ is the Prandtl number and $\nabla^2 = \nabla_h^2 + \partial^2/\partial z^2$ is the Laplacian operator with $\nabla_h^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$.

And

$$\tilde{f} = \exp\left[B\left(z-\frac{1}{2}\right)\right], \quad B = \left(\frac{\nu_{\max}}{\nu_{\min}}\right). \tag{11}$$

Normal modes can be defined by the perturbations

$$\left(w,\ T, C\right) = \left[W(z)\ ,\Theta(z)\ ,S(z)\right]\,e^{i(lx+my)} \tag{12}$$

Substituting the above expression into Equations (8) and (10), we obtain:

$$\tilde{f}\left(D^2-a^2\right)^2 W + 2D\tilde{f}\left(D^2-a^2\right)DW +$$
$$D^2\tilde{f}\left(D^2+a^2\right) = a^2\left(-R_T\Theta + R_S S\right)\left(1+\eta\,G(z)\right) \tag{13}$$

$$\left(D^2-a^2\right)\Theta + W = 0 \tag{14}$$

$$\frac{1}{Le}\left(D^2-a^2\right)S + S_r\left(D^2-a^2\right)\Theta + W = 0 \tag{15}$$

$$\left(D^2-a^2\right)\Theta_s = 0 \tag{16}$$

And boundary conditions are

$$W = D\Theta = DS = 0 \quad \text{at} \quad z = 1. \tag{17}$$

$$W = 0 = DC,\ \Theta = \Theta_s,\ D\Theta = k_r\,D\Theta_s \ \text{at}\ z = 0 \tag{18}$$

$$D\Theta_s = 0 \quad \text{at} \quad z = -d_r\,. \tag{19}$$

Here, $k_r = k_C/k_T$ is the thermal conductivity ratio, $d_r = d_s/d$ is the depth ratio. The solid-fluid interface becomes

$$D\Theta = k_r\,a\tanh\left(a\,d_r\right)\Theta. \quad \text{at } z = 0 \tag{20}$$

3. Method of Solution

By regular perturbation technique, the variables W,Θ and S are expanded in powers of a^2 in the form

$$\left(W,\Theta,S\right) = \sum_{i=0}^{N}\left(a^2\right)^i\left(W_i,\Theta_i,S_i\right) \tag{21}$$

Substitution of Eq. (21) into Eqs. $(13)-(15)$ and the boundary conditions and considering like powers of a^2, we get zeroth-order equations whose solutions are as follows

$$W_0 = S_0 = 0 = 0 \quad \text{and} \quad \Theta_0 = 1 \tag{22}$$

First-order equations are

$$D^4 W_1 + 2BD^3 W + B^2 D^2 W =$$
$$\left\{R - \frac{R_s}{Le}\right\}e^{-B\left(z-\frac{1}{2}\right)}\left(1+\lambda G(z)\right) \tag{23}$$

$$D^2\Theta_1 = 1 - W_1 \tag{24}$$

$$D^2 S_1 - 1 + S_r\left(D^2\Theta_1 - 1\right) = -LeW_1. \tag{25}$$

The boundary conditions are

$$W_1(1) = 0 = W_1(0) \tag{26}$$

$$D\theta_1(0) = k_r d_r\,\Theta_0(0) \tag{27}$$

The general solution of (23) for three types of gravity variation are

$$W_{1L}=\left\{R-\frac{R_s}{Le}\right\}\left[\begin{array}{l}c_1+c_2z+c_3e^{-Bz}+c_4ze^{-Bz}\\ +e^{-B\left(z-\frac{1}{2}\right)}\left(\frac{z^2}{2B^2}-\delta\right)\left(\frac{z^3}{6B^2}+\frac{z^2}{B^3}\right)\end{array}\right] \quad (28)$$

$$W_{1P}=\left\{R-\frac{R_s}{Le}\right\}\left[\begin{array}{l}c_5+c_6z+c_7e^{-Bz}+c_8ze^{-Bz}+\\ e^{-B\left(z-\frac{1}{2}\right)}\left(\frac{z^2}{2B^2}-\delta\right)\left(\frac{z^4}{12B^2}+\frac{2z^3}{3B^3}+\frac{3z^2}{B^2}\right)\end{array}\right] \quad (29)$$

$$W_{1C}=\left\{R-\frac{R_s}{Le}\right\}\left[\begin{array}{l}c_9+c_{10}z+c_{11}e^{-Bz}+c_{12}ze^{-Bz}+\\ e^{-B\left(z-\frac{1}{2}\right)}\left(\frac{z^2}{2B^2}-\delta\right)\left(\frac{z^5}{20B^2}+\frac{z^4}{2B^3}+\frac{5z^3}{B^4}+\frac{36z^2}{B^5}\right)\end{array}\right] \quad (30)$$

The compatibility condition is derived from equations (24) and (25)

$$\int_0^1\left[2+S_r\,Le-(1+Le)W_1\right]dz=-k_rd_r\left(1+S_r\,Le\right) \quad (31)$$

W_1 is substituted in the equation (31) and the critical Rayleigh number is obtained for distinct gravity functions.

In the limit $B\to 0$, which is the case of a constant-viscosity fluid layer over a solid plate of finite thermal conductivity and thickness, the expression of critical Rayleigh number reduced to

$$R=320\left(1+k_r\,d_r\right) \quad (32)$$

As $k_r\to 0$ or $d_r\to 0$ can be reduced much further to the result (Nield [48 and 49]), which are well-known ones, achieved previously.

4. Results and Discussion

In this paper, the problem of double-diffusive convective motion in a horizontal fluid layer with variable viscosity with gravity fluctuations due to the boundary solid slab is studied. Three different types of gravity fluctuations force are considered: the linear variation $G(z)=-z$, the parabolic variation $G(z)=-z^2$, and the cubic variation $G(z)=-z^3$. The influence of the solute Rayleigh number, the viscosity parameter, the Lewis number, the depth ratio, the gravity variation parameter, the Soret parameter, and the thermal conductivity ratio on the stability of the configuration studied. and outcomes are presented in Table1 and Figs. 2 to 13. The regular perturbation technique is used to solve the eigenvalue problems of linear theory. The following conclusions are taken from the above study.

The perturbed vertical velocity W for distinct values of thermal conductivity ratio and depth ratio for all three cases of gravity fluctuations are shown graphically in Figs 2,3 and 4. It is seen that the thermal profile becomes quadratic in vertical z-coordinate in the fluid layer as the values of k_r and d_r increases and velocity flow has maximum in the fluid layer in the lower part.

Figures 5,6,7 and 8 demonstrate the onset critical Rayleigh number with gravity parameter for distinct fixed depth ratio and thermal conductivity ratio for gravity fluctuations considered. It is noted that with increasing gravity parameter, the value of R^c also increases with higher values of thermal conductivity ratio and depth ratio. This is due to an increase in gravity parameter declines in the gravity force strength. Because of the recurrence of the structural disturbance as the center of gravity falls, the convective wave's onset is delayed. Additionally, it has been found that the stability of the structure is more consistent for linear gravity fluctuation whereas the system is less stable for cubic gravity variation(see Table1).

The effect of the viscosity variation parameter with three types of gravity fluctuations on the stability of the structure is shown in Fig. 9 and Figure 10. It is shown in the figures that the critical Rayleigh number first rises with the viscosity variation parameter, reaches a maximum, and then falls with a further rise in the value of B, resulting in the separation of three areas as seen in the isothermal boundary scenario (see Stengel et al. [16]).

Figure 11 depicts how the solute Rayleigh number and linear gravity variations affect the stability of the structure. It is noted that from the figure the critical Rayleigh number increases as the gravity parameter increases. Further, it is also noticed that the configuration is most steady Rs = 20 when compared to Rs = 0. Figure 12 demonstrates the effect of Lewis number Le on the stability mechanism with linear gravity fluctuations case for distinct fixed thermal conductivity ratio and depth ratio. It can be observed that the critical Rayleigh number falls down when Le rises. When heat diffusivity overcomes mass diffusivity and hence lags convection. Figure 13 displays the deviance of critical Rayleigh number as a function of λ for linear type gravity variation for distinct values of Sr. Clearly from Fig.13, R^c increases when δ increases and thus Sr stabilizes the configuration in the stationary mode.

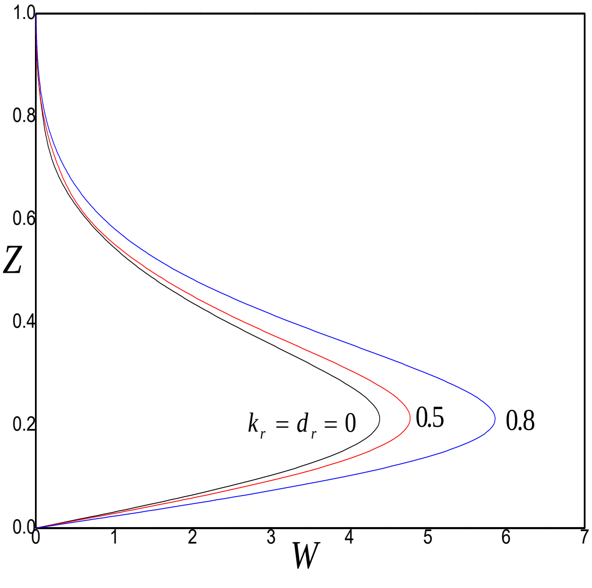

Fig. 2. Plot of W versus Z for distinct values of thermal conductivity ratio and depth ratio for linear gravity fluctuation with $R_S = 50, Sr = 0.5,$ and $Le = 0.5$.

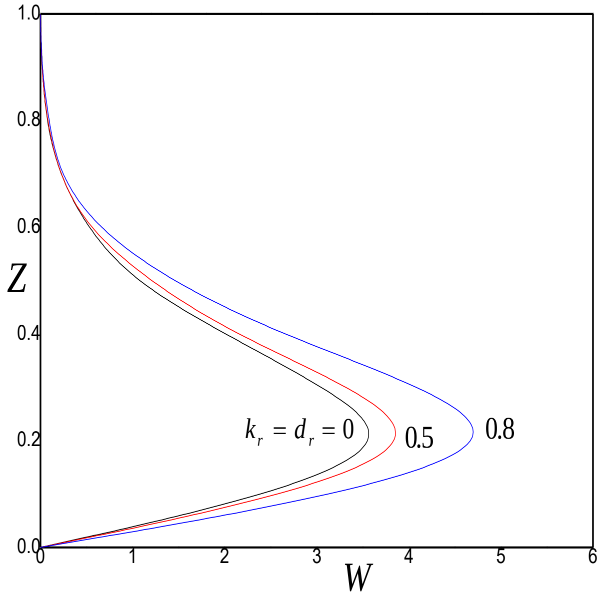

Fig. 3 Plot of W versus Z for distinct values of k_r and d_r for parabolic gravity fluctuation with $R_S = 50,$ $Sr = 0.5,$ and $Le = 0.5.$

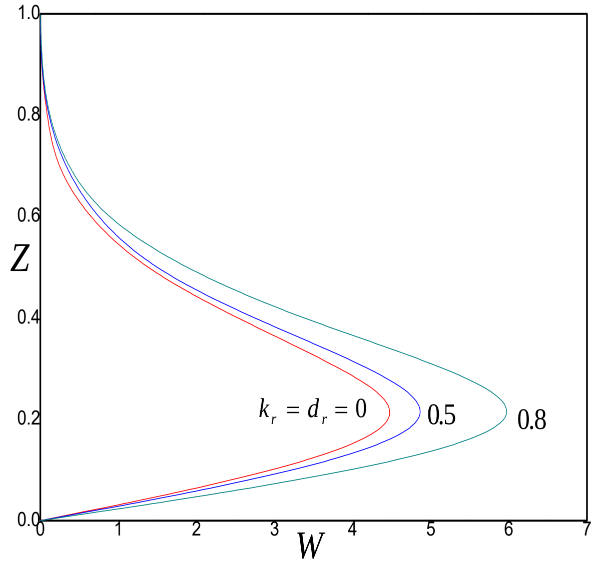

Fig. 4. Plot of W versus Z for distinct values of k_r and d_r for cubic gravity fluctuation with Rs =50, Sr =0.5 and $Le = 0.5.$

λ	**Critical Rayleigh number** (R^c)		
	G(z) = -z	G(z) = -z^2	G(z) = -z^3
0	780	760	730
0.5	820	770	740
1.0	960	778	752
1.5	1200	784	768
2	1500	796	784

Table 1 R^c and λ for three types of gravity variation with $Pe = 5$, $B = 5$, $Rs = 50$, $k_r = 0.5 = d_r$ and $Le = 1$.

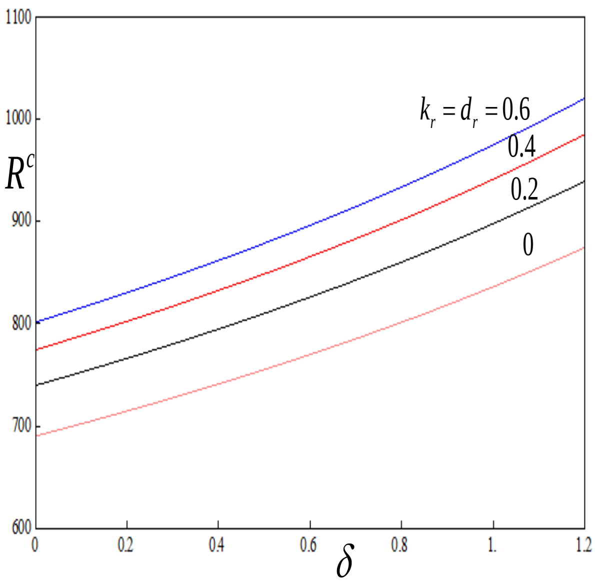

Fig.5 R^c versus δ with $R_S = 50$, $Sr = 0.5$, $Le = 0.5$, and $B = 10$ for distinct values of thermal conductivity ratio and depth ratio for linear gravity fluctuation.

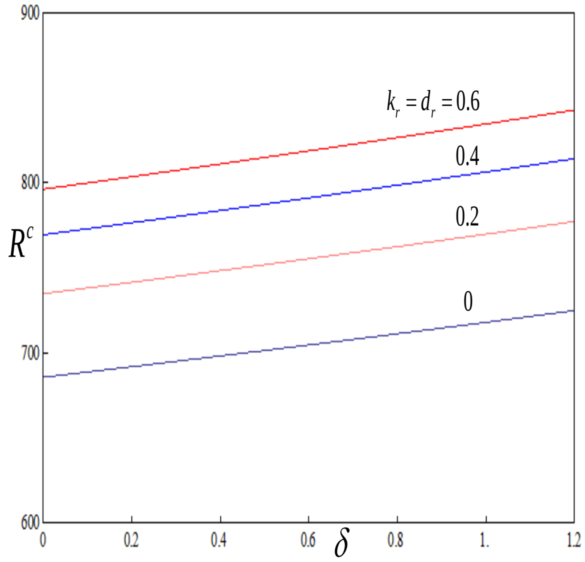

Fig.6 R^c versus δ with $R_S = 50$, $Sr = 0.5$, $Le = 0.5$, and $B = 10$ for distinct values of thermal conductivity ratio and depth ratio for parabolic gravity fluctuation.

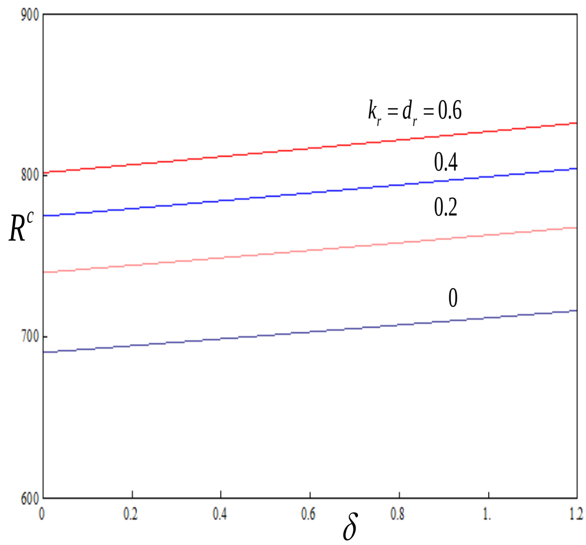

Fig.7 R^c versus δ with $R_S = 50$, $Sr = 0.5$, $Le = 0.5$, and $B = 10$ for distinct values of of k_r and d_r for cubic gravity fluctuation.

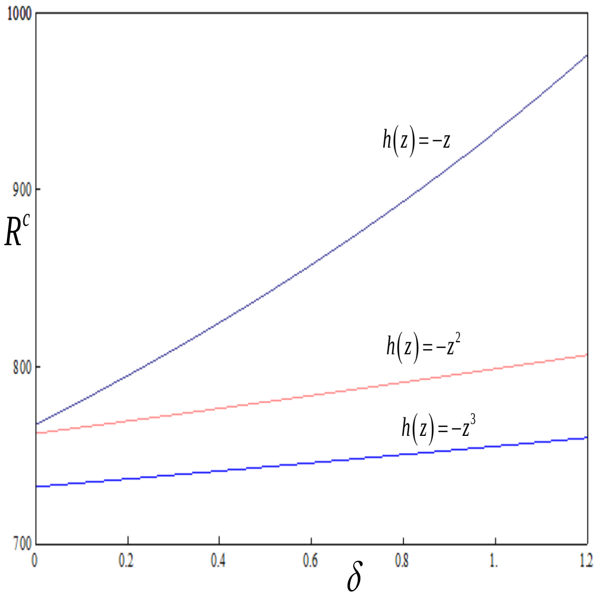

Fig.8 R^c versus δ with $R_S = 50$, $Sr = 0.5$, $k_r = 0.5$, $d_r = 0.5$, $Le = 0.5$, and $B = 10$ for three types of gravity fluctuations.

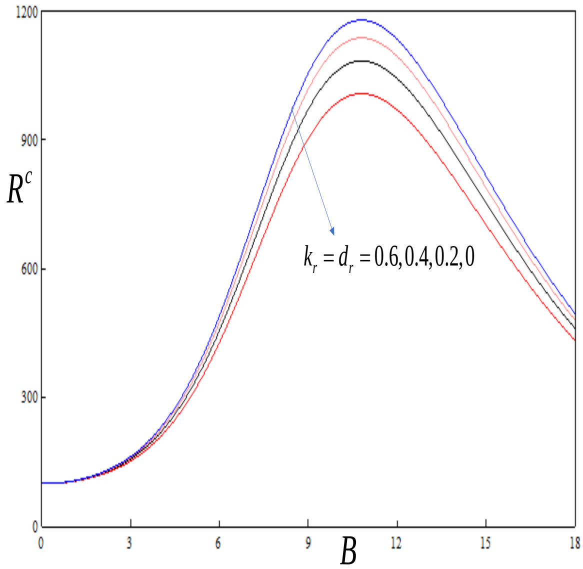

Fig.10 R^c versus B with $R_S = 50$, $Sr = 0.5$ and $Le = 0.5$ for different values of k_r and d_r for parabolic gravity field.

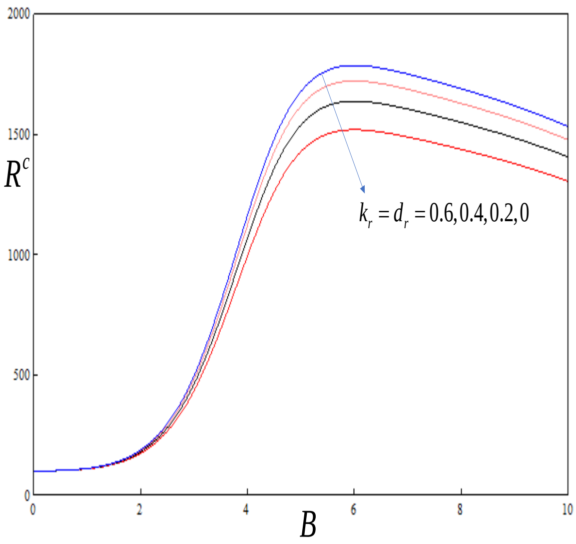

Fig.9 R^c versus B with $R_S = 50$, $Sr = 0.5$ and $Le = 0.5$ for different values of k_r and d_r for linear gravity field.

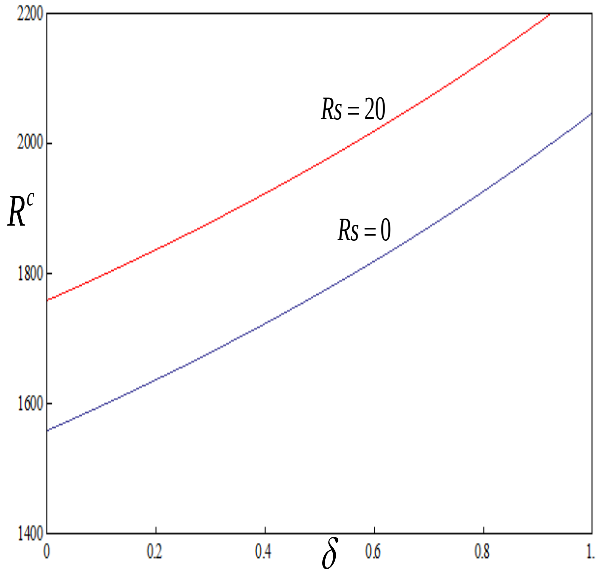

Fig.11 R^c versus δ with $Sr = 0.5$, $Le = 0.5$, and $B = 10$ for distinct values of R_S for linear gravity fluctuation.

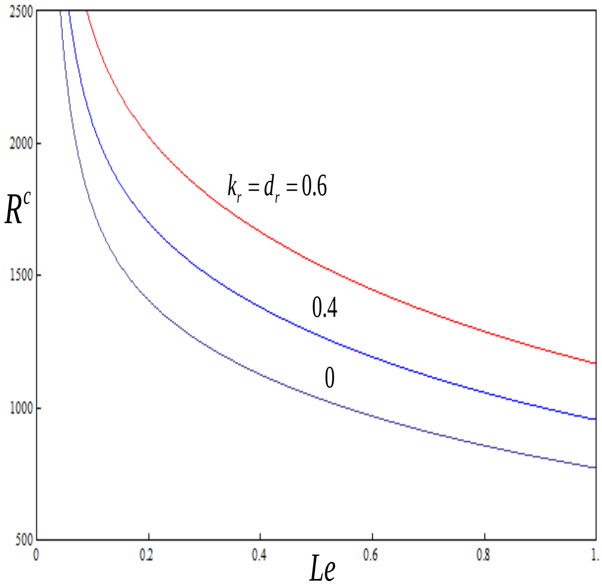

Fig.12 R^c versus Le with $R_S = 50$, $Sr = 0.5$, and $B = 10$ for distinct values of k_r and d_r for linear gravity fluctuation.

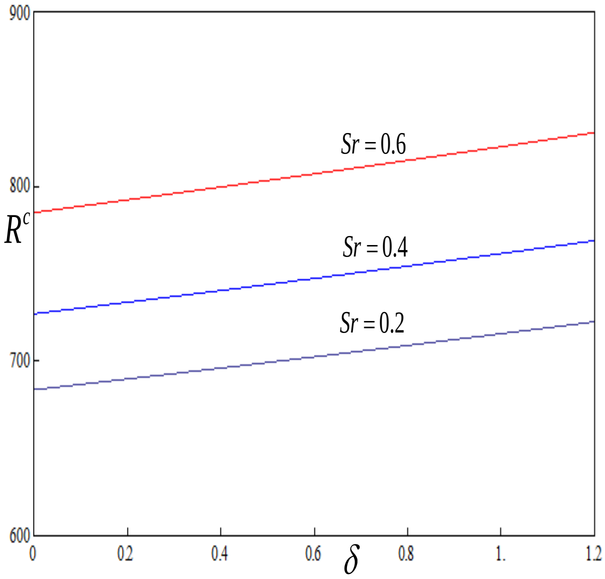

Fig.13 R^c versus δ with $R_S = 50$, $Le = 0.5$, $k_r = 0.5$, $d_r = 0.5$, and $B = 10$ for distinct values of Sr for linear gravity fluctuation.

5. Conclusions

In this study, the problem of the onset of double-diffusive convection in a fluid layer with the combined effects of changeable viscosity, Soret parameter, and gravity fluctuations is studied using linear stability analysis. Three distinct kinds (linear, quadratic, and cubic) of gravity field fluctuations are considered for the study. The key outcomes of the study are as follows:

- As the depth ratio and thermal conductivity ratio grow, the vertical velocity flow increases to its maximum.
- It has been discovered that the effects of increasing λ, B, Sr, k_r, dr and Rs arriving at lag convection. While Le is responded to enhance the start of convective motion.
- As the influence of the gravity field parameter, thermal conductivity ratio, depth ratio, Soret parameter, and solutal Rayleigh number increases, the size of the convective cells diminishes, while the viscosity parameter and Lewis number have a dual character on the dimension of convection cells.
- It has been found that for the linear type gravity fluctuation, the flow is more stable, and for cubic type gravity fluctuation, the flow is less stable.

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