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Research Article

Combined impact of variable viscosity and throughflow effects on the onset of convection in an anisotropic porous layer

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ABSTRACT

In the present article, the combined impact of vertical throughflow and temperature-reliant viscosity on the fluid-saturated anisotropic porous matrix is considered for investigation numerically by the Galerkin technique. The temperature-reliant viscosity is known to be exponential. The porous matrix is subject to continuous vertical throughflow. A parametric analysis is conducted by adjusting the following parameters: throughflow parameter, viscosity parameter, mechanical anisotropic parameter, and anisotropic thermal parameter. The findings reveal that the impacts of raising the viscosity parameter, downward throughflow parameter, and anisotropic thermal parameter delay the beginning of convection, whereas increasing mechanical anisotropic parameter and upward throughflow parameter destabilizes the porous system.

1. Introduction

Convective motion in the porous matrix was the subject of prominence study because of its significant applications. Some of these applications are, for example, assessing the performance of fiber insulations, groundwater flow prediction in aquifers, and nuclear engineering. A good review on this topic has been presented in [1–4].

Temperature-induced variations in viscosity and density can have a significant impact on the convection stability thresholds in fluid and porous media layers, as reported in depth in [5–8] and the references therein. The heat reliance of the fluid properties can modify the stream behavior in streams with heat move: specifically, its dependability attributes are grounded. The thickness shows a fairly articulated temperature variety for the greater part of the commonsense fluids since consistency is more heat-safe than thermal conductivity. Rossby[9] calculated that 20% to 25% of thermal conductivity and water viscosity values find that the kinematic viscosity parameter differs

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Nomenclatur

a	horizontal wave number	d	thickness of the porous layer
D	differential operator d/dz	R	Rayleigh number
$\mu(T)$	variable viscosity	p	pressure
B	viscosity parameter	T_0	temperature at the interface
W	perturbed vertical velocity	T	temperature
Pe	throughflow parameter	ξ	mechanical anisotropic parameter
β	slip parameter	$\vec{V}$	velocity vector (u, v, w)
∇_h^2	horizontal Laplacian operator	η	thermal anisotropic parameter
∇^2	Laplacian operator	$\vec{K}$	permeability tensor vector
ε_T	ratio of thermal diffusivities	ϕ	porosity of the porous medium
κ	thermal diffusivity	θ	amplitude of perturbed temperature
μ	fluid viscosity	ρ_0	fluid density
σ	temperature dependent surface tension	ν	kinematic viscosity

between 20% and 25% by approximately 10%, whereas just 1.5% of water thermal conductivity fluctuates. Torrance and Turcotte[10] found that that as temperature increases, liquid thickness decreases, while gases demonstrate a converse example. Many researchers have considered the effect of thickness changing temperature in convection problems in recent years(Barletta and Nield[11], Solomatov and Barr[12] and Booker[13], Shivakumara et al.[14], Suma et al.[15], Gangadharaiah ([16],[17]), and Gangadharaiah et al.[18],[19])).

The surface-driven penetrative convective motion in a composite system with exponential viscosity variance was reported by Gangadharaiah [20]. Ananda et al. [21] have investigated the penetrative convective motion with variable viscosity in a porous bed. Impact of exponential viscosity variation on Marangoni convective motion in a superposed system with a thin slab investigated by Gangadharaiah and Ananda[22]. The main objective of this study is to analyze the combined effects of exponential type viscosity variance and throughflow on the convective motion in an anisotropic porous matrix. Through normal Galerkin techniques, numerical findings are derived from the predominant equations. The outcomes of various relevant convection arrival parameters have been presented in detail.

2. Conceptual Model

Fig. 1 demonstrates the physical structure of the current study. The horizontal, isotropic porous matrix bounded between planes at $z=0$ & $z=d$ with continuous constant throughflow of vertical velocity W_0 and downward gravity $g(z)$. We

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presume that the viscosity depends exponentially on the temperature of the form $\mu = \mu_0 \, \exp\left[-A\left(T - T_0\right)\right]$.

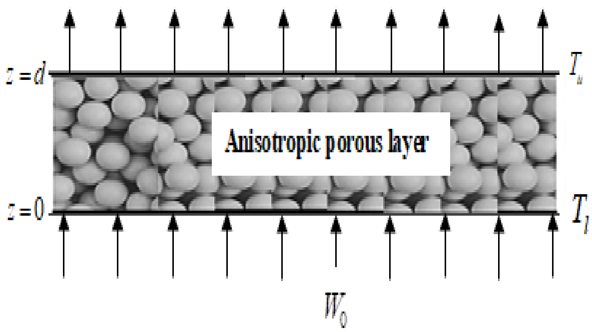

Fig. 1 Physical configuration

3. Mathematical Formulation

The mathematical governing relation for the above configuration are

$$\nabla \cdot \vec{V} = 0 \qquad (1)$$

$$0 = -\nabla p - \frac{\mu(T)}{\vec{K}} \cdot \vec{V} + \rho_0 \left[1 - \beta\left(T - T_0\right)\right] g \qquad (2)$$

$$A\frac{\partial T}{\partial t} + \left(\vec{V} \cdot \nabla\right) T = k_m \nabla^2 T \qquad (3)$$

where $\vec{K} = K_x\left(\hat{i}\,\hat{i} + \hat{j}\,\hat{j}\right) + K_x\left(\hat{k}\,\hat{k}\right)$ is permeability tensor, $\vec{V}$ is the velocity vector, $\kappa_T = \kappa_{Tx}\left(\hat{i}\,\hat{i} + \hat{j}\,\hat{j}\right) + \kappa_{Tz}\left(\hat{k}\,\hat{k}\right)$ is thermal diffusivity tensor, and $\mu(T)$ is variable viscosity.

The basic state is quiescent and is of the form

$$\left(u\ ,v, w\ ,p\ ,T\right) = \left(0\ ,0\ ,w_0, p_b(z), T_b(z)\right) \qquad (4)$$

Then, the basic temperature field is:

$$\frac{d^2 T_b}{dz^2} - w_0 \frac{dT_b}{dz} = 0 \qquad (5)$$

On solving Eq.(5), we get

$$T_b(z) = \frac{e^{Pe} - e^{Pe\,z}}{e^{Pe} - 1} \qquad (6)$$

From Fig.2, it is seen modulation of throughflow the $T_b(z)$ is to merely modifies the distribution within the porous bed. The basic state is slightly perturbed using the relation given by

$$\vec{V} = W_0\hat{k} + \vec{V}',\ p = p_b(z) + p',\ T = T_b(z) + \theta \qquad (7)$$

Applying Eq. (7) into Eqs. $(1)-(3)$, the linear stability equations become:

$$f(z)\nabla^2 w + \frac{1}{\xi} f'(z_m)\frac{\partial w}{\partial z} = R\nabla_{hm}{}^2 T' \qquad (8)$$

$$\left(A\frac{\partial}{\partial t} + Pe\frac{\partial}{\partial z} - \eta\nabla^2\right)T_m = w\left[\frac{Pe\ e^{Pe\,z}}{\left(1 - e^{Pe}\right)}\right] \qquad (9)$$

where

$$f(z) = \exp\left[B\left(z - \frac{1}{2}\right)\right], \quad B = \left(\frac{\nu_{\max}}{\nu_{\min}}\right).$$

thermal diffusivity tensor, and $\mu(T)$ is variable viscosity.

The basic state is quiescent and is of the form

We assume the solution is of the form

$$\left(w,\ T\right) = \left[W(z)\ ,\Theta(z)\right] e^{i(lx+my)} \qquad (11)$$

Substituting Eq. $(11)-$ into Eqs. $(8)-(9)$, we arrive

$$f(z)\left(D^2 - a^2\right)w + \frac{1}{\xi} f'(z) Dw = -R\,a^2\Theta \qquad (12)$$

$$\left(D^2 - PeD - \eta a^2\right)\Theta = W\left[\frac{Pe\ e^{Pe\,z}}{\left(1 - e^{Pe}\right)}\right] \qquad (13)$$

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where $R = \alpha g_0 \left(T_l - T_u\right) d^3 / \nu \kappa$ is the Rayleigh number.

The boundary positions are:

$$W = \Theta = 0 \quad at \quad z = 0, 1 . \tag{14}$$

3. Method of Solution

We now employ the Galerkin weighted residuals procedure to solve the system of Eqs. (12) and (13). Consequently ,W & Θ are considered as

$$W = \sum_{i=1}^{n} A_i W_i , \ \Theta = \sum_{i=1}^{n} B_i \Theta_i \tag{15}$$

With trial functions

$$W_i = \Theta_i = \sin\left(iz\right) \tag{16}$$

Using the governing parameters $\left(Pe, B \ ,\xi ,\eta, \ a\right)$, the eigenvalue R can be obtained.

4. Results and Discussion

The effect of exponential viscosity variance with throughflow on the stability of moment of convection in an anisotropic porous bed is examined numerically. The resulting eigenvalue problem is solved using a Galerkin process. In the present analysis, the governing parameters considered are the viscosity parameter $\left(B\right)$, the mechanical anisotropic parameter $\left(\xi\right)$, thermal anisotropic parameter$\left(\eta\right)$, and throughflow parameter $\left(Pe\right)$. The consistency of the system is achieved in terms of R^c and a_c by referring to different values η, ξ B and Pe.

Fig. 3 illustrates the impact of throughflow on the porous convection, and it is noted that the R^c increases for downward throughflow and decreases for upward throughflow. Also, it is found that R^c rises with the rise in the viscosity parameter B. Fig. 4 indicate the significance of the viscosity parameter on the convective motion. It is noted that the R^c rises with the rise of viscosity term B. Further it is revealed that system gives more stable for downward throughflow.

Fig. 5 and 6 depict the variation of R^c with ξ, for various values of viscosity parameter B with $\xi = \eta = 0.5$, for $Pe = -1$ and $Pe = 1$, respectively. The figures illustrate that as the mechanical anisotropic parameter is increased, the R^c decreases. This is as a result of the fact that rising the K_x will result in the cell's size increasing while lower the value of K_z results in a bigger variation in temperature between the bottom and top plates as presented by Degan et al., [23]. Further, it is revealed that the system is more stable for downward throughflow.

Fig. 7 demonstrates the effect of η for various values of B. It should be emphasized that with rising the value of η and viscosity parameter B , the R^c also increases. Rising η causes a slowdown in κ_{Tz}, as a result, the heat movement vertically through it slows.

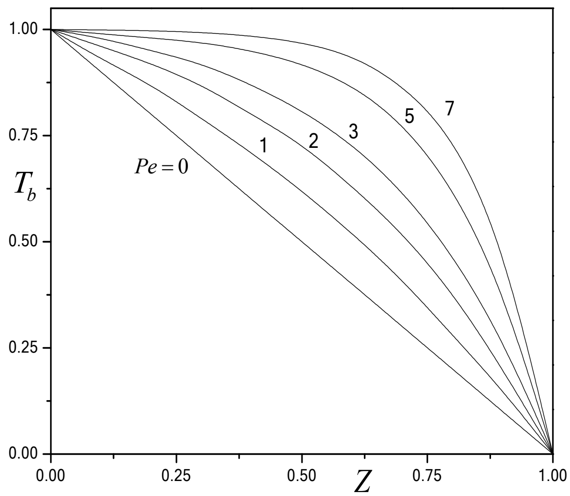

Fig. 2 Plot of the basic state temperature distributions for different values of $Pe = 5$.

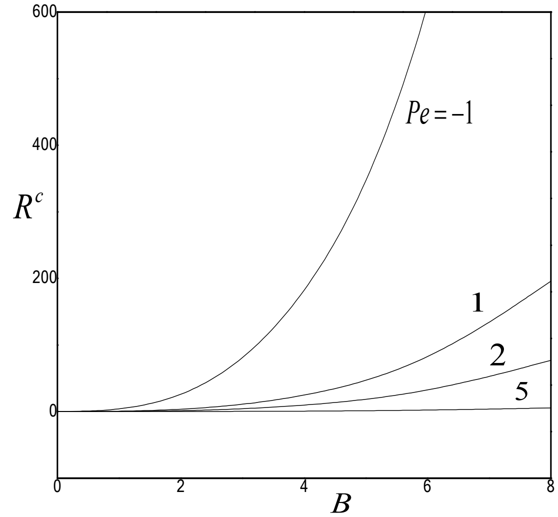

Fig.4 R^c versus B with $\xi = \eta = 0.5$, for different values of Pe.

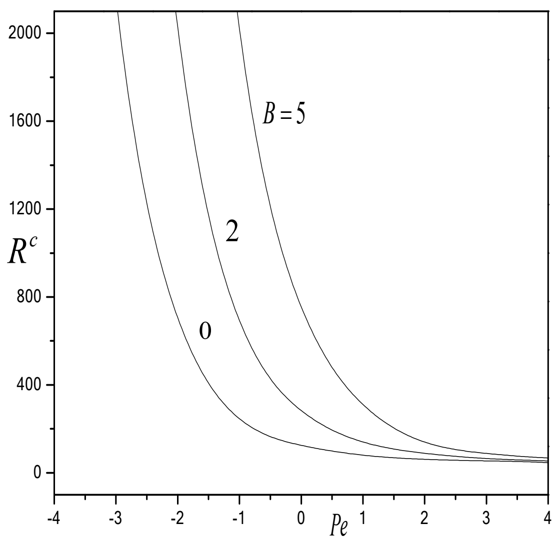

Fig.3 R^c versus Pe with $\xi = \eta = 0.5$, for different values of B.

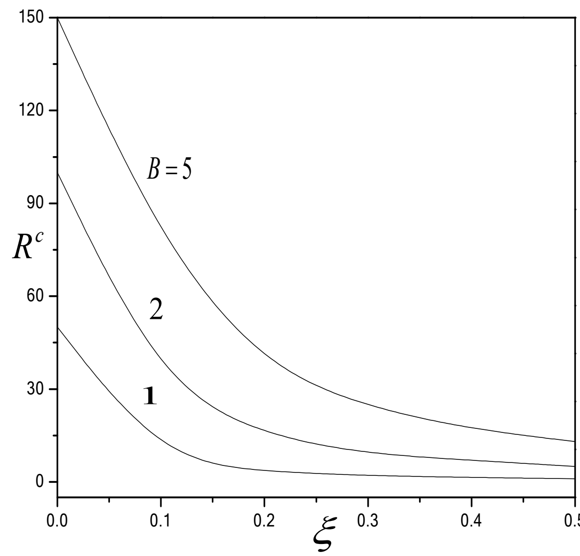

Fig.5 R^c versus ξ for $\xi = \eta = 0.5$, with $Pe = -1$ for different values of B.

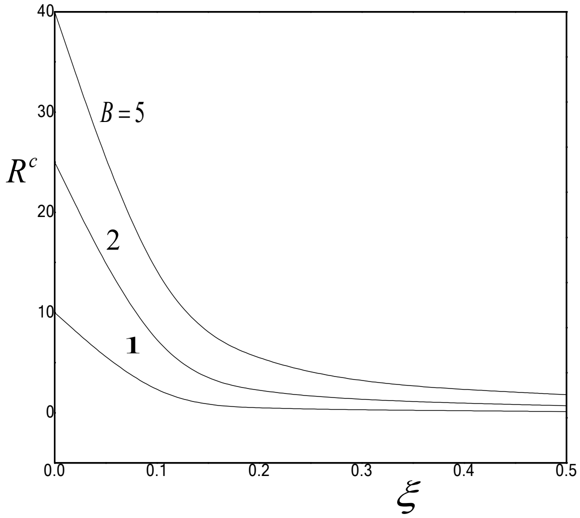

Fig.6 R^c versus ξ for $\eta = 0.5$ & $Pe = 1$ for different values of B .

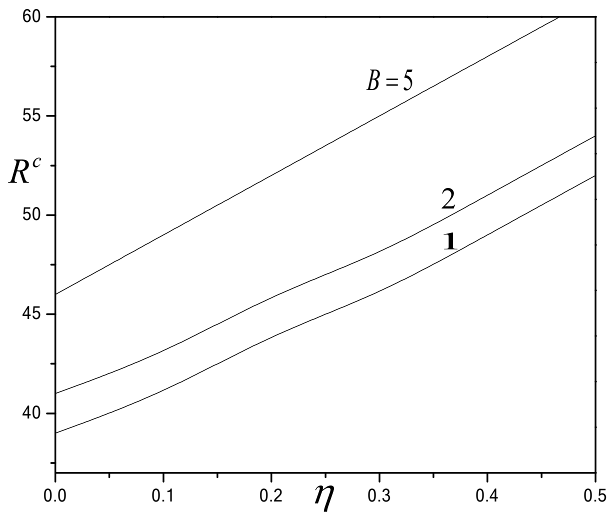

Fig.7 R^c versus η for $\xi = 0.5$ & $Pe = 1$ for different values of B .

5. Conclusions

Numerical analysis of the appearance of convective unsteadiness in an anisotropic porous matrix with throughflow and exponential viscosity variance is studied by using linear stability via the Galerkin method. The findings reveal that the impacts of raising the of the viscosity parameter, downward throughflow parameter, and anisotropic thermal parameter delay the beginning of convection and increasing mechanical anisotropic parameter and upward throughflow parameter advance the beginning of convection. Hence by increasing the values of η, B and by decreasing the values of ξ and Pe , convection in an anisotropic porous layer with adjusting throughflow can be delayed, and hence the system can be stabilized.

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