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Research Article

MHD natural convection in nanofluid filled square cavity with isothermally heated hexagonal block

Rowsanara Akhter*, [1], Mohammad Mokaddes Ali [2]

[1] *Department of Natural Science, The International University of Scholars, Dhaka-1212, Bangladesh*
[2] *Department of Mathematics, Mawlana Bhashani Science and Technology University, Tangail-1902, Bangladesh*

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ABSTRACT

This article explores the influence of natural convection heat transfer in a square enclosure with hexagonal block filled by Al_2O_3-H_2O nanofluid in presence of external magnetic field. The two vertical walls of the enclosure are kept at constant cold temperature, T_c, and the top horizontal wall is insulated whereas the remaining walls of the enclosure are maintained high temperature, T_h. The governing equations are formulated using Boussinesq approximations and solved numerically employing built-in-finite element method and then presented graphically in terms of streamlines, isotherms, average Nusselt number and average temperature. It is found that the different values of physical parameters: Rayleigh number ($Ra = 10^3$ -10^6), nanoparticle volume fraction ($\phi = 1\%$ -5%) and Hartman number ($Ha = 0$-100) affects the streamlines, isotherm contours and heat transfer rate. Comparison of the present results with previously published results on the basis of special cases has been performed and found to be a good agreement.

1. Introduction

Natural convection in closed enclosure frequently occurs in many industrial and engineering applications such as ventilation and thermal insulation systems, cooling of electronic devices, heat exchangers, solar collectors and nuclear reactor, energy storage and conservations etc. Considering this importance many researchers have carried out a number of theoretical and experimental studies on natural convection flow in different geometries. Basak et al. [1] performed a numerical study on natural convection in a triangular enclosure for different thermal boundary conditions. Their results showed that heat transfer rate and flow circulations with temperature distributions influenced by the different thermal boundary conditions. It was also noted that heat transfer rate and flow strength increased significantly for increasing of Rayleigh number and Prandtl number. The problem of natural convection in a uniformly and linearly heated square cavity was investigated by Sathiyamoorthy et al. [2] and found the flow and heat transfer characteristics affected by the heated walls and natural convection parameter. The effect of volumetric

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**Corresponding e-mail*: dr.rowsanara@ius.edu.bd (Rowsanara Akhter)

heat source on natural convection in a wavy enclosure was numerically investigated by Oztop et al. [3] and concluded that the flow and heat transfer affected noticeably with the amplitude of sinusoidal walls and Rayleigh number.

Moreover, natural convection in presence of magnetic field has been studied to regulate the flow and heat transfer rate. Sathiyamoorthy and Chamkha [4] investigated the variation of local and average Nusselt number for natural convection in presence of magnetic field effect and found the local and average Nusselt number reduced for the effect of magnetic field. Ali et al. [5] studied the effect of magnetic field on double diffusive natural convection in a square cavity. They recommended that heat transfer rate decreased for increasing effect of magnetic field. Gajbhiye and Eswaran [6] investigated the influence of magnetic field on Rayleigh-Benard convection in constricted enclosure. Their results reflected that Nusselt number reduces for increasing Hartmann number.

In recent years, researchers have developed a new class of heat transfer fluids named as nanofluid to overcome the limitations of traditional heat transfer fluids which have been being used extensively in various fields of engineering where improved heat transfer is required. A mathematical model was developed by Khanafer et al. [7] to investigate the fluid flow and heat transfer phenomena of nanofluids in a two-dimensional enclosure. Their results confirmed that the heat transfer rate increases for suspended nanoparticles and the structure of fluid flow altered as well. Later on, Tiwari and Das [8] extended the study [7] for mixed convection in two-sided lid-driven heated square cavity and observed that both the fluid flow and heat transfer affected with the direction of the lid walls and solid volume fraction of nanoparticles. After that, Yu et al. [9] conducted a similar study [7] for transient natural convection in a differentially heated square cavity. Their results indicated the flow development and time-averaged Nusselt number are independent on solid volume fraction of nanoparticles. Ali et al. [10] performed a numerical analysis to present the fluid flow and heat transfer behavior of natural convection in a nanofluid filled hexagonal enclosure and identified that the flow pattern, temperature distribution and heat transfer rate varied with governing parameters. Later on, Akhter et al. [11] numerically optimized the heat transfer rate due to natural convection parameter with higher volume fraction in a porous cavity. Kazi et al. [12] numerically analyzed the effects of magnetic field on mixed convection flow in a vented cavity with volumetric heat generation or absorption in presence of cylindrical obstacle and found highest heat transfer rate for both increasing Reynolds and Prandtl numbers in absences of magnetic field effect. Islam et al. [13] conducted a numerical study of free convection flow and heat transfer in a nanofluid filled right-angled triangular cavity in presence of magnetic field and demonstrated that temperature transfer value decreases with the enhancement of Hartmann number, and increases buoyancy driven parameter. Later on, Molli and Naikoti [14] employed finite element technique to investigate magnetohydrodynamic free convection nanofluids flow over a flat plate saturated by porous medium and observed that skin-friction and Nusselt number values of Cu-water nanofluid are higher than pure fluid.

In addition, separated obstacle of different shapes is used to control the fluid flow behaviors due to convective heat transfer in enclosures (closed or open), which is encountered where the fluid flow become restricted or bifurcated. In this regard the inclined edges of hexagonal block can have an important effect on flow and heat transfer behaviors as compared to block of other shapes. The main objective of this study is to investigate the flow and temperature characteristics of natural convection in an Al_2O_3-water nanofluid filled square cavity with a centered hexagonal block in presence of magnetic field effect. To the best of author's knowledge, no such study has been conducted yet. The computed results are presented in terms of streamlines, isotherms, average Nusselt numbers and average temperature and discussed in the following section.

2. Physical Model

Natural convection flow is considered in a differentially heated square cavity which is filled with Al_2O_3-water nanofluid. A hexagonal block is placed at the center of the cavity. The bottom wall of the cavity and also the side walls of the hexagonal block are heated at high temperature T_h while the vertical walls are cooled at temperature T_c. The *x*-axis is considered along the horizontal direction while *y*-axis is normal to it and the gravitational acceleration is performed in the vertically downward direction. A uniform external magnetic field is imposed to the horizontal direction. The thermo physical properties of nanoparticles and water have been given in Table 1. The physical configuration of this study is shown in Fig. 1.

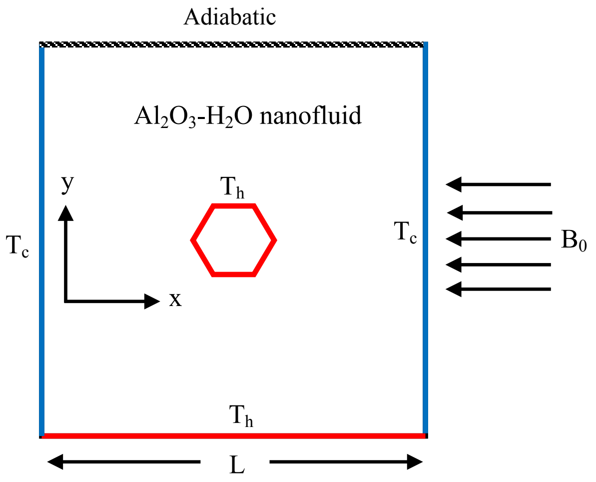

Fig. 1 Schematic diagram of the physical system.

Table 1: Thermo-physical properties of the base fluid and alumina [15, 16].

Physical properties	Base fluid (water, 300K)	Al_2O_3
$C_p\ (J/kgK)$	4179	765
$\rho(kg/m^3)$	997.1	3970
$k(W/mK)$	0.613	40
$\beta(1/K)$	2.1×10^{-4}	0.85×10^{-5}
$\sigma(\Omega.m)^{-1}$	0.05	10^{-10}

3. Mathematical Analysis

The mathematical model of this investigation has been developed by using the Boussinesq approximation and given in non-dimension form as follows:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \qquad (1)$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\rho_f}{\rho_{nf}}\frac{\partial P}{\partial X}+\frac{\nu_{nf}\Pr}{\nu_f}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) (2)$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\rho_f}{\rho_{nf}}\frac{\partial P}{\partial Y}+\frac{\nu_{nf}\Pr}{\nu_f}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)$$
$$+\left(\frac{\beta_{nf}}{\beta_f}\right)Ra\Pr\theta-\left(\frac{\rho_f}{\rho_{nf}}\right)\left(\frac{\sigma_{nf}}{\sigma_f}\right)Ha^2\Pr V \qquad (3)$$

$$U\frac{\partial\theta}{\partial X}+V\frac{\partial\theta}{\partial Y}=\frac{\alpha_{nf}}{\alpha_f}\left(\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}\right) \qquad (4)$$

The following dimensionless variables have been used to non-dimensionalize the above equations:

$X = x/L$, $Y = y/L$, $U = u\ L/\alpha_f$, $V = vL/\alpha_f$, $P = pL/\rho_f\alpha_f^2$ and $\theta=(T-T_c)/(T_h-T)$ (5)

Where $\Pr = \nu_f\ /\alpha_f$ is the Prandtl number and $Ra=g\beta_f(T_h-T_c)L^3/\nu_f\alpha_f$ is the Rayleigh number.

The corresponding boundary conditions are:

at all the solid boundaries: U = 0, V = 0; (6a)
at the vertical walls: $\theta = 0$; (6b)
at the bottom horizontal walls: $\theta = 1$; (6c)
at the top horizontal walls: $\frac{\partial\theta}{\partial y}=0$; (6d)
at the boundaries of inner obstacle: $\theta = 1$. (6e)

In order to focus the heat transfer rate in terms of local Nusselt number, the local Nusselt number in dimensional form for the heated surface is defined as

$$\bar{N}u = -\frac{k_{nf}}{k_f}\frac{\partial T}{\partial x} \qquad (7)$$

and the average Nusselt number along that wall of the enclosure is also defined as

$$Nu=(1/L)\int_0^1 \bar{N}u\,dY \qquad (8)$$

The average temperature of the fluid domain in the cavity is written as

$\theta_{av} = \int(\theta/\bar{V})\ d\bar{V}$, where $\bar{V}$ is the volume of the enclosure. (9)

3.1. Computational Procedure

The computation procedure has been carried out by using the Galerkin weighted residual finite element method and the solution domain has been discretized with non-uniform triangular elements, which has been well described by Taylor and Hood [17] and Dechaumphai [18]**.** The detail application of this method is available in Ref. [19].

3.2. Grid Sensitivity Test

The grid sensitivity test h been performed by using five types of meshes. Comparing the presented numerical results in the Table 1 and Fig. 2, it can be concluded that the grid size of 13149 nodes and 25838 provides accurate solution of the present problem.

3.3. Mesh Generation

To complete the simulation procedure, the solution domain is subdivided into a set of sub-domains; called finite elements. The values of the dependent variable are computed at the vertices each element. The meshing of the solution domain is presented in Fig.3.

Table 2: Grid sensitivity test at Pr = 6.2, Ra = 10^3, Ha = 50 and ϕ = 1%. (Elem.→Elements)

Nodes (Elem.)	Ns:3358 Es:6486	Ns: 5758 Es: 11056	Ns:8358 Es:15946
Nu_{av}	5.19783	5.82575	6.07045
Nodes (Elem.)	Ns: 13149 Es: 25838	Ns:14520 Es: 28810	
Nu_{av}	6.10082	6.11231	

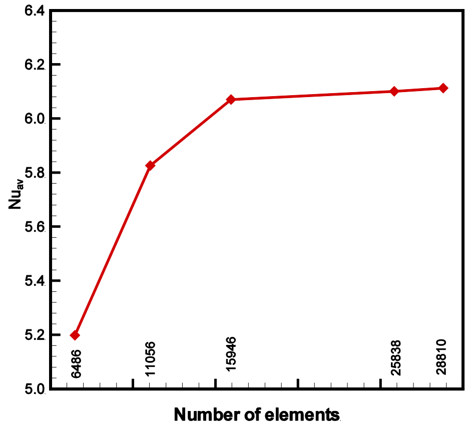

Fig. 2. Grid refinement test.

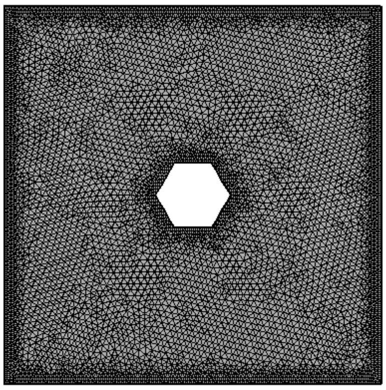

Fig. 3. Mesh size of the schematic diagram.

3.4. Code validation

The present code has been validated using comparison between the present results and the results of Pirmohammadi et al. [20]. Pirmohammadi et al. [20] studied natural convection flow in a differentially heated cavity in presence of magnetic field by using finite volume code. We have simulated similar geometrical and mathematical model of Ref. [20] by using finite element code in order to justify the present numerical procedure. It is important to note that finite element method provides more accurate results than finite volume methods and it is more suitable for irregular geometries than others methods. A satisfactory agreement has been reflected in Table 3 which points out the accuracy of the present simulation.

4. Results and discussion

In this section, the numerical results of physical quantities in term of streamlines, isotherms and average Nusselt number for the different values of governing parameters are illustrated in Fig. 4 and Fig. 5, respectively. Though the discussion is concise, the findings of this analysis provide more information regarding fluid flow and heat transfer enhancement.

The influence of magnetic field on streamlines and isotherms are illustrated in Fig.4 (a) and 4 (b), respectively with different Ra and ϕ while Pr = 6.2. From Fig. 4 (a), it can be observed that, there developed two circulations beside the central hexagonal cylinder where the left one is rotated in clockwise direction and right one is in counter clockwise direction. Fluid moves up near the middle of the bottom wall and also beside the block and then down from top to bottom near the side walls due to free convection heat transfer causing for thermal conditions. In Fig. 4(a), significant variations are found in central circulation cells with increasing Ha whereas the boundary cells are unaltered with greater Ha and take a form like spectacle shaped. Moreover, the strength of flow filed decreases for increasing of magnetic effect. The reason behind it's that external magnetic field effect creates Lorentz's force which retards the fluid motion. In addition, greater effect of buoyancy force ($Ra=10^6$) increases the magnitude of fluid velocity and the shapes of stream function are affected remarkable due the dominance of convection heat transfer. Furthermore, the strength of flow filed decreases for increasing volume fraction of nanoparticles in base fluid. Because suspended nanoparticles increase the density of nanofluid. In order to give a better understanding of flow circulations inside the enclosure, we have surveyed the numerical results of stream function within the cavity which reflects the fluid current increases with increasing Rayleigh number and reduces for increasing of magnetic strength and volume fraction of nanoparticles. On the other hand, in Figs. 4(b), high temperature region observed near the hot walls and the isotherm contours are almost similar with increasing Ha at low value of Ra and ϕ. Moreover, isotherms are more condensed near to the corners of the bottom wall which indicates the dominance of conduction mode heat transfer and the gradient of isotherm contours increases near the hexagonal block and bottom wall for higher Ra which is an indication of the dominance of convection mode heat transfer. In addition, onion shaped isotherm contours are found for all values of Ha at low value of Ra and ϕ which are converted in mushroom shape for the higher Ra at all values of Ha. Furthermore, the effect of Ra on the variation of isotherms is shrinked with greater volume fraction due to the presence of nanoparticles.

Fig. 5(a) and (b), represent average Nusselt number and average temperature for varying of Hartmann number Rayleigh number and volume fraction. In From 5(a), average Nusselt number declines for increasing Ha with fixed Ra and ϕ whereas it increases significantly due to increase in Ra and ϕ for all Ha. Because, greater effect of magnetic field produces temperature inside the enclosure whereas the greater buoyancy force accelerates fluid movement and the addition of nanoparticles increases the thermal conductivity of nanofluid. Moreover, from the numerical observation, it is found that the heat transfer rate increases by 67.37% while Ra varies from 10^3 to 10^6 in absence of magnetic field and 40.29% while Ra varies from 10^3 to 10^6 at higher value of Ha (Ha =100). In addition, the existence of nanoparticles augments average Nusselt number by 21.85% while Ha = 0 and 14.98% while Ha = 100.

5. Conclusion

In this paper, the effect of magnetic field on natural convection flow in an alumina-water nanofluid filled cavity with a heated hexagonal has been investigated numerically. The numerical results of streamlines, isotherms and average Nusselt number have been computed for a range of Ha, Ra and ϕ while Pr is fixed at 6.2. The finding can be summarized as follows:

- Fluid flow field within the enclosure is influenced significantly due to the effect of magnetic field, Rayleigh number and solid volume fraction of nanoparticles.
- Temperature distribution inside the enclosure is influenced noticeably for the effect of Rayleigh number, and minutely for the effect of magnetic field and solid volume fraction of nanoparticles.
- Heat transfer rate increases rapidly for the greater effect of Rayleigh number and volume fraction of nanoparticles with lower effect of magnetic field.
- Heat transfer rate increases by 67.37% while Ra varies from 10^3 to 10^6 in absence of magnetic field (Ha = 0) and it becomes 40.29% at Ha =100.
- The best augmentation of average Nusselt number is found for 5% Vol. which is 21.85% at Ha = 0 and 14.98% at Ha = 100 compare to pure water.

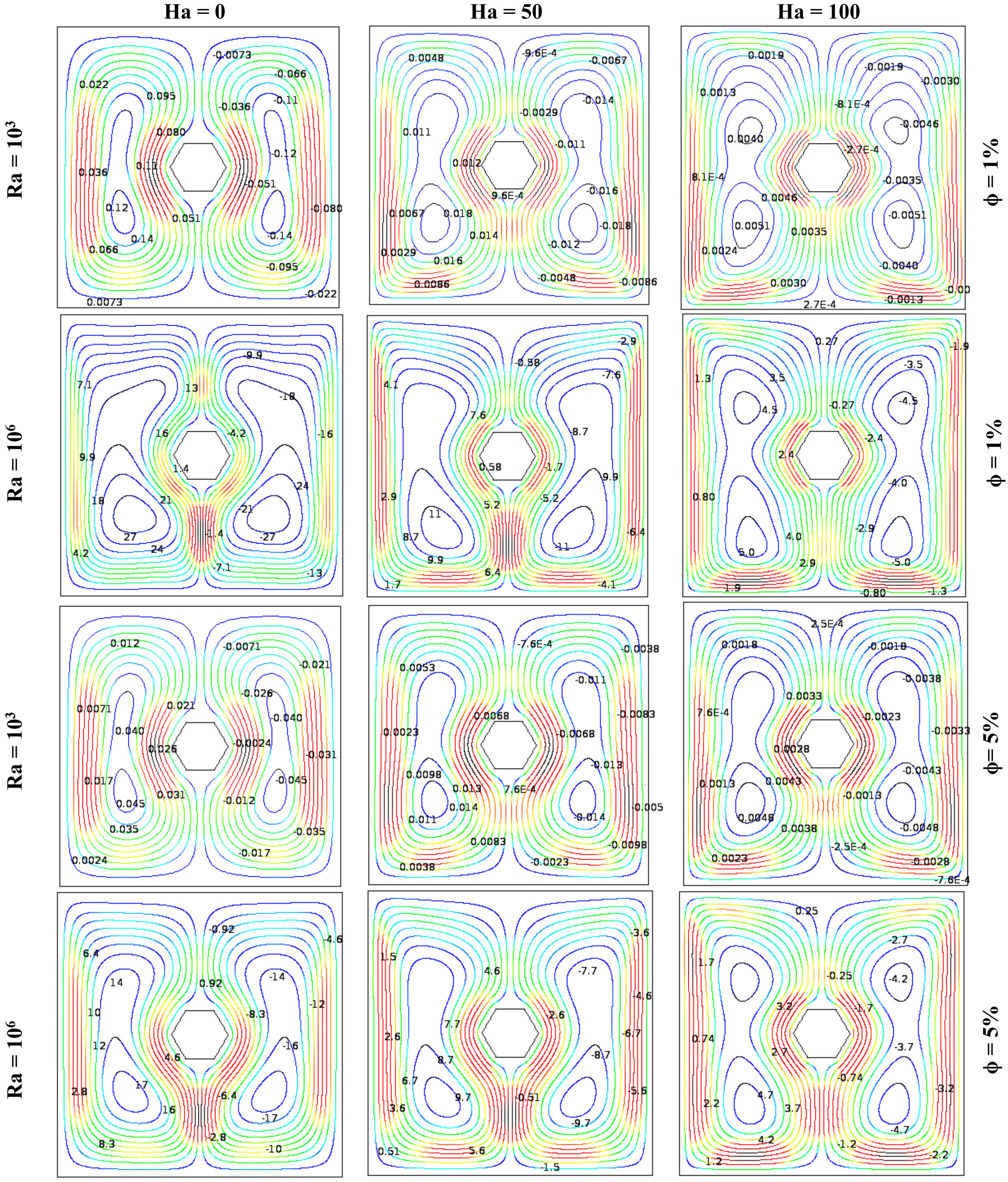

Fig.4 (a). Streamlines for different Hartmann number, Rayleigh number and volume fraction while Pr = 6.2.

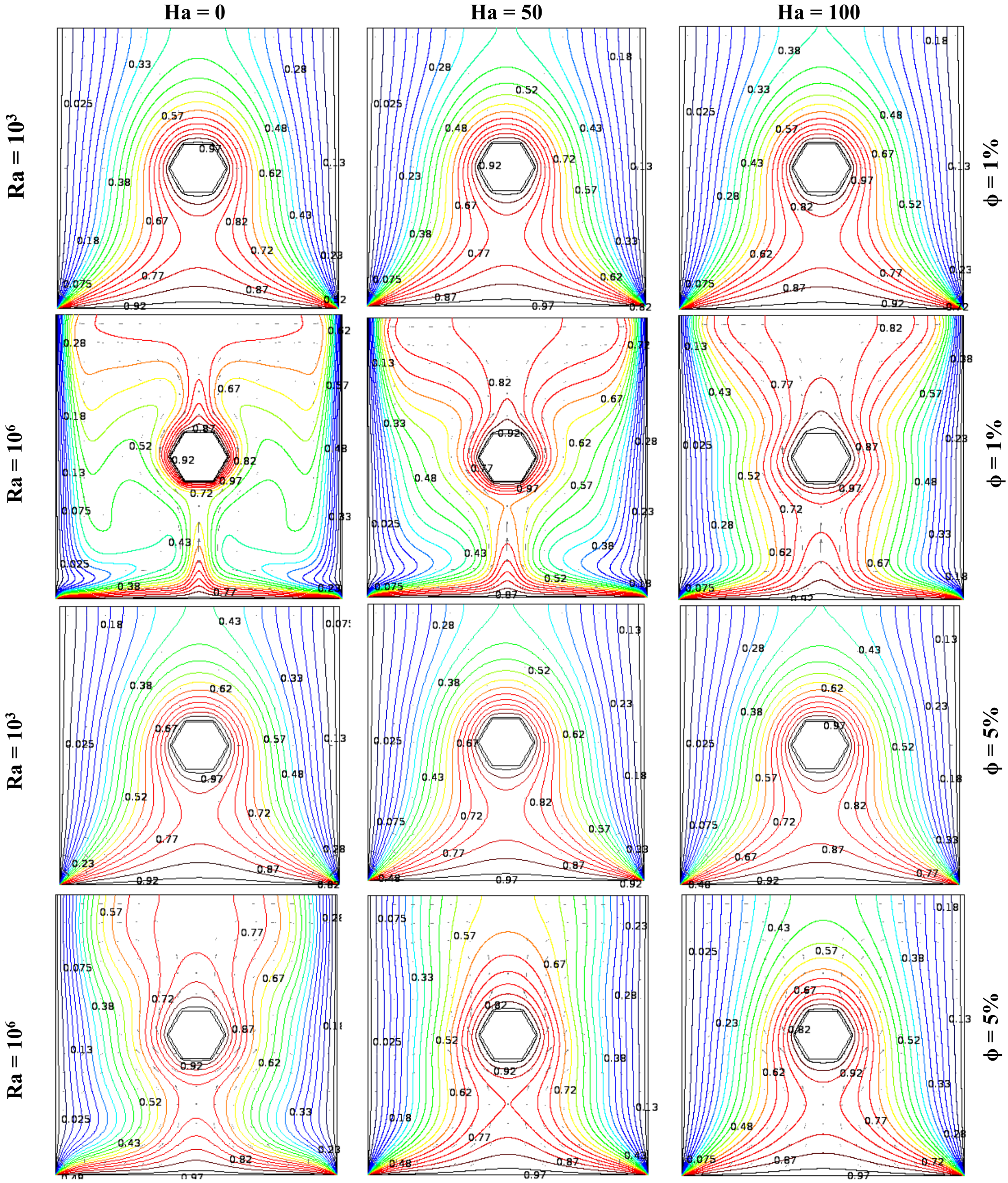

Fig.4 (b). Isotherms for different Hartmann number, Rayleigh number and volume fraction while Pr = 6.2.

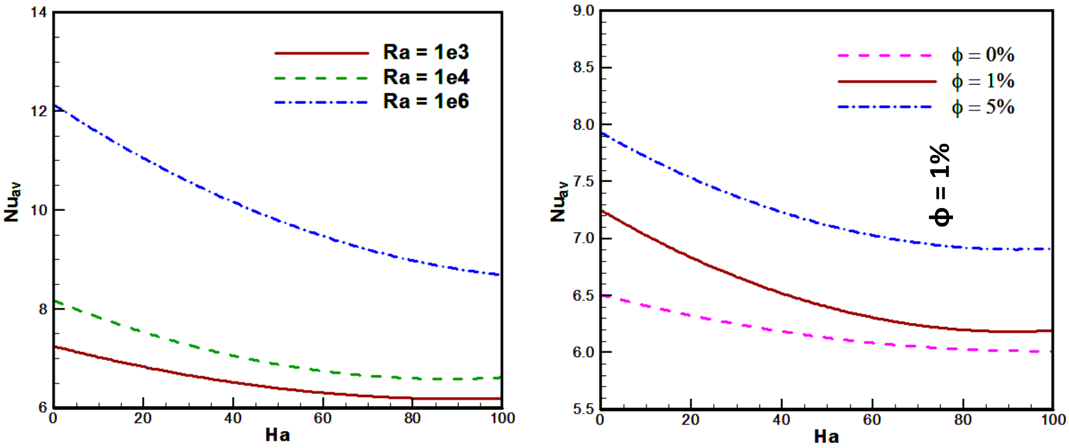

Fig. 5. Average Nusselt number for different Hartmann number, Rayleigh number and volume fraction while Pr = 6.2.

Table 3: Comparison of average Nusselt number for various Rayleigh number and Hartmann number.

Ra	Ha	Pirmohammadi et al. [20]	Present study (With similar geometrical and mathematical model of Ref. [20])	Error (%)	Present study
10^4	0	2.29	2.245	2.0	6.77464
	10	1.97	1.928	2.2	6.62515
	50	1.06	1.037	2.2	6.18245
	100	1.02	1.003	1.7	6.11888

References

[1] T.Basak, S. Roy, C.Thirumalesha, Finite element analysis of natural convection in a triangular enclosure: Effects of various thermal boundary conditions, Chemical Engineering Science62(9)(2007)2623- 2640.

[2] M. Sathiyamoorthy, T .Basak, S. Roy, I. Pop, Steady natural convection flows in a square cavity with linearly heated side wall(s), International Journal of Heat and Mass Transfer 50 (2007) 766-775.

[3] H F. Oztop, E Abu-Nada, Y. Varol, A. Chamkha, Natural convection in wavy enclosures with volumetric heat sources, International Journal of Thermal Sciences, 50 (4) (2011) 502-514.

[4] M. Sathiyamoorthy, A. J. Chamkha, Natural convection flow under magnetic field in a square cavity for uniformly (or) linearly heated adjacent walls, International Journal of Numerical Methods for Heat & Fluid Flow 22(5) (2012) 677-698.

[5] M. M. Ali, M. A. Alim, R.Nasrin, Effect of magnetic field on double diffusive natural convection flow in a closed chamber using nanofluid, AIP Conference Proceedings 1980 (1) (2018) p-050020

[6] N. L. Gajbhiye, V. Eswaran, Numerical simulation of MHD flow and heat transfer in a rectangular and smoothly constricted enclosure, International Journal of Heat and Mass Transfer 83 (2015) 441-449.

[7] K. Khanafer, K. Vafai,M. Lightstone, Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids, International Journal of Heat and Mass Transfer 46 (2003) 3639-3653.

[8] R. K. Tiwari, M. K. Das, Heat transfer augmentation in a two-sided lid- driven differentially heated square cavity utilizing nanofluids, International Journal of Heat and Mass Transfer50 (2007) 2002-2018.

[9] Z. T. Yu, W. Wang, X Xu, L. W. Fan, Y. CHu, K. F. Cen, A numerical investigation of transient natural convection heat transfer of aqueous nanofluids in a differentially heated square cavity, Int. Communications in Heat and Mass Transfer 38 (2011) 585-589.

[10] M. M. Ali, M. A., Alim, R. Akhter, S. S. Ahmed, MHD natural convection flow of CuO/water nanofluid in a differentially heated hexagonal enclosure with a tilted square bloc, International Journal of Applied and Computational Mathematics3(1) (2017) 1047-1069.

[11] R. Akhter, M. M. Ali, M. A. Alim, Hydromagnetic Natural Convection Heat Transfer in a Partially Heated Enclosure Filled with Porous Medium Saturated by Nanofluid, International Journal of Applied and Computational Mathematics 5(3) (2019) 52.

[12] M. R. Kazi, M. M. Ali, M. Hakim, M. Rahman, Magnetic Mixed Convection in a Vented Cavity with Volumetric Heat Generation or Absorption in Presence of a Cylindrical Obstacle, International Journalof Thermofluid Science and Technology 8(3) (2021) 080304.

[13] T. Islam, N. Parveen, M. F. A. Asad, Hydromagnetic natural convection heat transfer of copper-water nanofluid within a right-angled triangular cavity, International Journal of Thermofluid Science and Technology, 7(3) (2020) 070304.

[14] S. Molli, K. Naikoti, MHD natural convective flow of Cu-water nanofluid over a past infinite vertical plate with the presence of time dependent boundary condition, International Journalof Thermofluid Science and Technology, 7(3) (2020) 1-18.

[15] K. C. Lin, A. Violi, Natural convection heat transfer of nanofluids in a vertical cavity: Effects of non-uniform particle diameter and temperature on thermal conductivity, International Journal of Heat and Fluid Flow 31(2) (2010) 236-245.

[16] T. Armaghani, M. S. Sadeghi, A. M. Rashad, M. A. Mansour, A. J. Chamkha, A. S. Dogonchi, H. A. Nabwey, MHD mixed convection of localized heat source/sink in an Al2O3-Cu/water hybrid nanofluid in L-shaped cavity, Alexandria Engineering Journal 60(3) (2021) 2947-2962.

[17] C. Taylor, P. Hood,A numerical solution of the Navier-Stokes equations using finite element technique, Computer and Fluids1 (1973) 73-89.

[18] P. Dechaumphai, Finite Element Method in Engineering, 2nd ed. Chulalongkorn University Press, Bangkok 1999.

[19] M. M. Ali, R. Akhter, R., M. A. Alim, MHD natural convection and entropy generation in a grooved enclosure filled with nanofluid using two-component non-homogeneous model, SN Applied Sciences 2(4) (2020) 1-25.

[20] M. Pirmohammadi, M, Ghassemi, G. A. Sheikhzadeh, Effect of a Magnetic Field on Buoyancy-Driven Convection in Differentially Heated Square Cavity, IEEE Transactions on Magnetics 45 (2009) 407-411.