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Research Article

Double-diffusive penetrative convection in a fluid overlying a porous layer

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ABSTRACT

In the present study, the commencement of double-diffusive convection with an internal heat source is studied using a linear instability analysis. The system consists of a fluid layer on top of a porous layer saturated with the same fluid. The boundaries are insulating to temperature perturbations, and the regular perturbation technique is applied to obtain the Rayleigh number. The results of detailed stability characteristics are presented for crucial physical factors, such as thermal Rayleigh number, the inverse Lewis number, depth ratio, the solute Rayleigh number, and heat source strength.

1. Introduction

Double-diffusive convection, which depicts convection driven by two separate density gradients, has sparked many research activities in recent years because of its broad range of applications. Some unique areas of application include the growth of metal crystals, solar ponds, insulation of buildings and equipment, energy storage and recovery, geothermal energy extraction and reservoirs, dispersion of pollutants in the environment, the underground disposal of nuclear wastes and material and food processing (Nield [1], Straughan [2]). Among the most recent contributions are (Capone et al. [3], Chaya and Gangadharaiah [4], Malashetty and Biradar [5], Malashetty et al. [6], Gangadharaiah et al. camse [7], Chang [8], Hill and Carr [9], Hill and Straughan [10]).

Convective motion in composite layers due to volumetric heating has attracted immense attention in the current past because of its prevalence in energy-related and geophysics engineering problems, including underground disposal of radioactive waste materials, heat removal from nuclear fuel debris, storage of food-stuff, exothermic chemical reactions in the packed-bed reactor and so on. Recent contributions include (Carr [11], Suma et al. [12], Khalili et al. [13], Gangadharaiah et al. [14], Shivakumara et al. [15], Gangadharaiah[16], Gangadharaiah and Ananda [17], Straughan[18], and Gangadharaiah and Suma[19]).

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Nomenclature

a	horizontal wave number	d	thickness of the porous layer
D	differential operator	d_m	thickness of the porous layer
$\vec{g}$	acceleration due to gravity	Q	heat source in the fluid layer
Q_m	heat source in the porous layer	Ns_m	dimensionless heat source in the porous layer
Ns	dimensionless heat source in the fluid layer	η_m	inverse porous Lewis number for porous layer
Da	Darcy number	p	pressure
R_T	Thermal Rayleigh number	T_0	temperature at the interface
R_S	Solute Rayleigh number	η	inverse fluid Lewis number for porous layer
W	perturbed vertical velocity	T	temperature
Pr	Prandtl number for fluid layer	Pr_m	Prandtl number for porous layer
β	slip parameter	$\vec{V}$	velocity vector (u, v, w)
∇_h^2	horizontal Laplacian operator	Ms	Solute Marangoni number
∇^2	Laplacian operator	$\vec{V}$	velocity vector
ε_T	ratio of thermal diffusivities	ϕ	porosity of the porous medium
κ	thermal diffusivity	θ	amplitude of perturbed temperature
μ	fluid viscosity	ρ_0	fluid density
σ	temperature dependent surface tension	ν	kinematic viscosity

In this study, we combine a two-layer system consisting of a layer of fluid superimposing a porous layer saturated with the same fluid (e.g., Sheng and Chen et al. [20], Saleem et.al. [21], Gangadharaiah [22] and Princewill [23], with double-diffusive convection.

A lot of study attention in terms of single-layer models, both liquid and porous (Hill [24, 25], Malashetty and Biradar [5], Gangadharaiah et al. [26]). Chang [27] considers this situation in the context of fluid floating on top of porous media. This article advances the fluid-porous model by allowing for double-diffusive convection for the first time. Double-diffusive Marangoni convection in a two-layer system is studied by Gangadharaiah [22]. Double-diffusive convection induced by selective absorption of radiation in a two-layer system reviewed by Princewill [23] and recently double-diffusive surface driven convection in a fluid-porous system is studied by Gangadharaiah [28]. In this work, double-diffusive convection in composite layers with an internal heat source is checked using a linear instability analysis. The analytical solution that is obtained is analyzed by varying the governing parameters.

2. Conceptual Model

We consider the horizontal two-layer system of an anisotropic porous bed of width d_m underlying a fluid layer of width d, the lower boundary of the porous layer is taken to be rigid(see Fig.1).

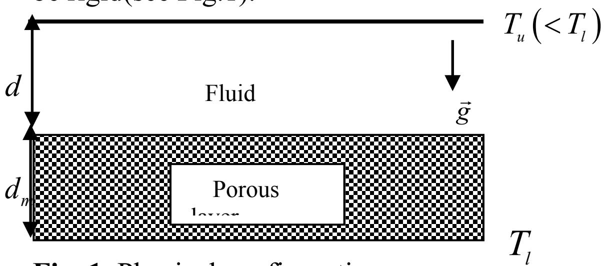

Fig. 1. Physical configuration

3. Mathematical Formulation

The mathematical governing relation for the above configuration are

3.1 Fluid zone

$$\nabla\cdot\vec{V}=0 \quad (1)$$

$$\rho_0\left(\frac{\partial\vec{V}}{\partial t}+\left(\vec{V}\cdot\nabla\right)\vec{V}\right)=\mu\nabla^2\vec{V}-\nabla p$$
$$-\vec{k}g\,\rho_0\left(1-\alpha_T\left(T-T_0\right)+\alpha_C\left(C-C_0\right)\right) \quad (2)$$

$$\frac{\partial T}{\partial t}+\left(\vec{V}\cdot\nabla\right)T=\lambda_T\nabla^2T+Q \quad (3)$$

$$\frac{\partial C}{\partial t}+\left(\vec{V}\cdot\nabla\right)C=\lambda_C\nabla^2C \quad (4)$$

3.2 Porous zone

$$\nabla_m\cdot\vec{V}_m=0 \quad (5)$$

$$\frac{\rho_0}{\phi}\frac{\partial\vec{V}_m}{\partial t}=-\nabla_m p_m-\frac{\mu}{K}\vec{V}_m$$
$$-\vec{k}\,g\rho_0\left(1-\alpha_T\left(T_m-T_0\right)+\alpha_C\left(C_m-C_0\right)\right) \quad (6)$$

$$A\frac{\partial T_m}{\partial t}+\left(\vec{V}_m\cdot\nabla_m\right)T_m=\lambda_{Tm}\nabla^2T_m+Q_m \quad (7)$$

$$\frac{\partial C_m}{\partial t}+\left(\vec{V}_m\cdot\nabla\right)C_m=\lambda_{Cm}\nabla^2C_m \quad (8)$$

In order to investigate the stability of the fundamental solution, infinitesimal disturbances are introduced.

$$\vec{V}=\vec{V}',T=T_b\left(z\right)+T'\;\&$$
$$C=C_b\left(z\right)+C',\,p=p_b(z)+p' \quad (9)$$

$$\vec{V}_m=\vec{V}'_m,T_m=T_{mb}\left(z\right)+T'_m\;,$$
$$C_m=C_{mb}\left(z\right)+C'_m,\,p_m=p_{mb}(z)+p'_m\,, \quad (10)$$

The dimensional less disturbance equations are given by(after linearization)

$$Pr^{-1}\frac{\partial\nabla^2w}{\partial t}=R_T\nabla_h^2\theta-R_S\nabla_h^2C+\nabla^4w \quad (11)$$

$$\left(-\nabla^2+\frac{\partial}{\partial t}\right)T=w\,f\left(z\right) \quad (12)$$

$$\left(\frac{\partial}{\partial t}-\eta\nabla^2\right)C=w \quad (13)$$

$$\left(\frac{1}{Pr_m}\frac{\partial}{\partial t}+\nabla_m^2\right)w_m=R_m\nabla_h^2\theta_m-R_{Sm}\nabla_h^2C_m \quad (14)$$

$$\left(A\frac{\partial}{\partial t}-\nabla^2\right)T_m=w_mf\left(z_m\right) \quad (15)$$

$$\left(\frac{\partial}{\partial t}-\eta_m\nabla^2\right)C=w_m \quad (16)$$

Where $\eta=\frac{1}{Le}$ is the inverse fluid, Lewis number $\eta_m=\frac{1}{Le_m}$ is the inverse porous Lewis number, $R_T=\frac{g\alpha_T\,d^3\rho_0\Delta T}{\lambda_T\mu}$ is the thermal Rayleigh number, $R_S=\frac{g\alpha_C\,d^3\rho_0\Delta C}{\lambda_T\mu}$ is the solutal Rayleigh number and

$$R_T=\frac{\zeta^6}{\varepsilon_T^2\varepsilon_C\delta^2}R_m\,,\;R_S=\frac{\zeta^6}{\varepsilon_T^2\varepsilon_C\delta^2}R_{Sm}$$

3.3 Normal mode analysis

$$\left(w,\,T,\,C\right)=\left[W\left(z\right),\Theta\left(z\right),S\left(z\right)\right]f(x,y) \quad (17)$$

$$\left(W_m,T_m,C_m\right)=\left[W_m\left(z_m\right),\Theta_m\left(z_m\right),S\left(z_m\right)\right]\times f_m\left(x_m,y_m\right) \quad (18)$$

Where $f(x,y)$ and $f_m\left(x_m,y_m\right)$ are horizontal plan forms satisfying $\nabla_h^2f=-a^2f$ and $\nabla_{mh}^2f_m=-a_m^2f_m$.

Substituting Eqs. (17) - (18) in Eqs. (11) - (16), we obtain the following ordinary differential equations:

$$\left(D^2-a^2\right)^2W=R_T\Theta-R_SS \quad (19)$$

$$\left(D^2-a^2\right)\Theta=-Wf\left(z\right) \quad (20)$$

$$\eta\left(D^2-a^2\right)S=-W \quad (21)$$

$$\left(D_m^2-a_m^2\right)W_m=R_m\Theta_m-R_{Sm}S_m \quad (22)$$

$$\left(D_m^2-a_m^2\right)\Theta_m=-W_mf\left(z_m\right) \quad (23)$$

$$\eta_m\left(D_m^2-a_m^2\right)S_m=-W_m \quad (24)$$

The boundary conditions are of the form

$$W=D\Theta=DS=0\;\text{ at }\;z=1 \quad (25)$$

$$DW=0\,at\,z=1 \quad (26)$$

$$D_m\Theta_m = D_m S_m = W_m = 0 \; at \; z_m = -1 \tag{27}$$

and $at \;\; z = 0,$ are

$$W = \frac{\zeta}{\varepsilon_T} W_m \tag{28}$$

$$D\Theta = D_m\Theta_m \tag{29}$$

$$DS = D_m S_m \tag{30}$$

$$\Theta = \frac{\varepsilon_T}{\zeta}\Theta_m \tag{31}$$

$$S = \frac{\varepsilon_s}{\zeta} S_m \tag{32}$$

$$\left[D^2 - 3a^2\right]DW = \frac{-\zeta^4}{\delta^2\varepsilon_T} D_m W_m \tag{33}$$

$$\left[D^2 - \frac{\alpha\zeta}{\delta} D\right]W = \frac{-\alpha\zeta^3}{\delta\varepsilon_T} D_m W_m \tag{34}$$

4. Solution by regular perturbation technique

The dependent variables are now expanded in powers of a^2 in the form

$$\left(W, \Theta, S\right) = \sum_{i=0}^{N}\left(a^2\right)^i \left(W_i, \Theta_i, S_i\right) \tag{35}$$

$$\left(W_m, \Theta_m, S_m\right) = \sum_{i=0}^{N}\left(\frac{a^2}{\zeta^2}\right)^i \left(W_{mi}, \Theta_{mi}, S_{mi}\right) \tag{36}$$

Substituting these equations in to obtained eigen value problem and collecting the leading order in a^2 become,

$$D^4 W_0 = 0 \tag{37}$$

$$D^2\Theta_0 = -W_0 f\left(z\right) \tag{38}$$

$$\eta D^2 S_0 = -W_0 \tag{39}$$

$$D_m^2 \, W_{m0} = 0 \tag{40}$$

$$D_m^2\Theta_{m0} = -W_{m0} f\left(z_m\right) \tag{41}$$

$$\eta_m D_m^2 S_{m0} = -W_{m0} \tag{42}$$

and the boundary conditions (25)–(34) become

$$W_0 = 0, D\Theta_0 = 0, \; DW_0 = 0 \; at \; z = 1 \tag{43}$$

$$W_{m0} = 0, D_m\Theta_{m0} = 0, \; at z_m = -1 \tag{44}$$

and $at \; z = 0,$ are

$$W_0 = \frac{\zeta}{\varepsilon_T} W_{m0} \tag{45}$$

$$\Theta_0 = \frac{\varepsilon_T}{\zeta}\Theta_{m0} \tag{46}$$

$$S_0 = \frac{\varepsilon_s}{\zeta} S_{m0} \tag{47}$$

$$D\Theta_0 = D_m\Theta_{m0} \tag{48}$$

$$DS_0 = D_m S_{m0} \tag{49}$$

$$D^2 W_0 - \frac{\alpha\zeta}{\delta} DW_0 = \frac{-\alpha\zeta^3}{\delta\varepsilon_T} D_m W_{m0} \tag{50}$$

$$D^3 W_0 = \frac{-\zeta^4}{\delta^2\varepsilon_T} D_m W_{m0} \tag{51}$$

The solution above equations is given by

$$W_0 = 0, \; \Theta_0 = \frac{\varepsilon_T}{\zeta}, \; S_0 = \frac{\varepsilon_S}{\zeta} \tag{52}$$

$$W_{m0} = 0, \Theta_{m0} = 1, \; S_{m0} = 1 \tag{53}$$

the first order in a^2, Eqs$\left(37\right)-\left(42\right)$ then reduce to

$$D^4 W_1 = \frac{1}{\zeta}\left(\varepsilon_T R_T - \varepsilon_S R_s\right) \tag{54}$$

$$D^2\Theta_1 - \frac{\varepsilon_T}{\zeta} = -W_1 f\left(z\right) \tag{55}$$

$$\eta D^2 S_1 - 1 = -W_1 \tag{56}$$

$$D_m^2 \, W_{m1} = \left(R_m - R_{Sm}\right) \tag{57}$$

$$D_m^2\Theta_{m1} - 1 = -W_{m1} f\left(z_m\right) \tag{58}$$

$$\eta_m D_m^2 S_{m1} - 1 = -W_{m1} \tag{59}$$

and the boundary conditions $\left(25\right)-\left(34\right)$ become

$$W_{m1} = 0, D_m\Theta_{m1} = 0, \text{at } z_m = -1 \tag{60}$$

$$W_1 = 0 = DW_1, D\Theta_1 = 0, \;\; \text{at } z = 1 \tag{61}$$

and $at \; z = 0,$ are

$$W_1 = \frac{\zeta}{\varepsilon_T} W_{m1} \tag{62}$$

$$\Theta_1 = \frac{\varepsilon_T}{\zeta}\Theta_{m1} \tag{63}$$

$$S_1 = \frac{\varepsilon_s}{\zeta} S_{m1} \tag{64}$$

$$D\Theta_1 = D_m\Theta_{m1} \tag{65}$$

$$DS_1 = D_m S_{m1} \tag{66}$$

$$D^2W_1-\frac{\alpha\zeta}{\delta}DW_1=\frac{-\alpha\zeta^3}{\delta\varepsilon_T}D_mW_{m1} \quad (67)$$

$$D^3W_1=\frac{-\zeta^4}{\delta^2\varepsilon_T}D_mW_{m1}. \quad (68)$$

Then solvability condition is given by

$$\begin{Bmatrix}\int_0^1 f(z)W_1\,dz+\frac{1}{\zeta^2}\int_{-1}^0 W_{m1}\,dz\\ +\\ \frac{1}{\eta}\int_0^1 W_1\,dz+\frac{1}{\eta_m\zeta^2}\int_{-1}^0 W_{m1}\,dz\end{Bmatrix}=\begin{Bmatrix}\frac{(\varepsilon_T+\varepsilon_s)}{\zeta}\\ +\\ \frac{(\eta+1)}{\zeta^2}\end{Bmatrix} \quad (69)$$

The general solution of Eqs. $(54)\&(57)$ are respectively given by

$$W_1=\frac{1}{\zeta}(\varepsilon_TR_T-\varepsilon_SR_S)\left(c_1+c_2z+c_3z^2+c_4z^3+\frac{z^4}{24}\right) \quad (70)$$

$$W_{m1}=(R_m-R_{Sm})\left(c_5+c_6z_m+\frac{z_m^2}{2}\right) \quad (71)$$

Where

$c_1=3\delta\varepsilon_T\ (2\ \varepsilon_T\eta\ \delta+\zeta\alpha\ ),$

$c_2=\left(-2\ \delta-3\zeta^2\alpha+\varepsilon_s\zeta^2\delta\eta\right)\Delta,$

$c_3=7\varepsilon_T\alpha\eta\delta(\zeta^3\text{-}\delta\eta-1\ ),$

$c_4=\dfrac{\varepsilon_T\zeta(\ 2\varepsilon_s\delta+\varepsilon_T\zeta\alpha)}{2\delta\Delta},$

$c_5=c_6=\dfrac{-6\varepsilon_T^2(\ 2\delta+\zeta\varepsilon_S\alpha)}{\zeta^2\ \Delta}$

$\Delta=6\ \zeta^2\delta\,R_S\ +\ \left(2\ \eta\varepsilon_T\text{-}\ 3\ R_S\delta\right)\zeta^3\alpha$

Substituting for W_1 and W_{m1} in Eq.(69), we obtain an expression for the solute Rayleigh number R_m in the form

$$R_m=\frac{\dfrac{720\varepsilon_T^2\varepsilon_C\delta^2}{\zeta^6}\begin{pmatrix}3\zeta^2\delta(-Rs+\varepsilon_S\eta_m)+\\ 3\alpha\delta(\varepsilon_T-\zeta^2\varepsilon_S\eta_m)+\alpha\zeta^3(1+\varepsilon_S\eta_m\varepsilon_T)\end{pmatrix}}{\Delta_1+\Delta_2} \quad (72)$$

Where

$\Delta_1=5\varepsilon_T\zeta^2\delta Rs_m+\left(2\eta\varepsilon_S\eta_m\varepsilon_T\text{-}3Rs_m\delta^2\right)\zeta^3,$

$\Delta_2=\left(2\eta^2\varepsilon_T+5Rs_m\delta^3\right)\zeta^3\alpha+6\zeta^4\delta Rs_m.$

5. Results and Discussion

The onset of penetrative convective motion in a two-layer system composed of a horizontal binary fluid with a porous layer has been examined. The perturbation technique is used to solve the resulting eigenvalue problem. We now present analytical findings obtained for the effects of Solutal Rayleigh number R_m, depth ratio ζ, volumetric heat source strengths $(Ns\,\&\,Ns_m)$, inverse fluid Lewis number η, and inverse porous Lewis number η_m. Throughout the calculations, we choose parameter values consistent with previous studies, namely $\varepsilon_T = 0.7$, $\varepsilon_S = 3.75$, $\alpha = 0.1$.

Setting $R_{Sm}=0$ and in the absence of internal heating ($Ns=0$), the known exact value $R^c=720$ (Sparrow et al. [29]) is retrieved for single fluid layer case $(\zeta>>1)$, and in the lack of internal heating ($Ns_m=0$), we recover the known exact value $R_m^c=12$ (Nield and Bejan [1]) for single porous layer case $(\zeta<<1)$.

To assess the effect of heat source strength, Fig. 2 depicts the impact of Solutal Rayleigh number R_m versus depth ratio ζ for two different values of heat source strengths $(Ns=Ns_m=0\,\&\,Ns=Ns_m=5)$ with $\eta_m=\eta=0.2\,\&\,Rs_m=0.2$. It should be observed that for the above-mentioned values heat sources, the Solutal Rayleigh number reaches higher values at lower values of ζ and internal heating in both layers hastens the onset of convection.

It is obvious to notice that the influence of the inverse fluid and porous Lewis numbers $\eta\,\&\,\eta_m$ on the fluid-porous layer mode. It is observed that from Fig.3 and Fig. 4, the Solutal Rayleigh number reaches higher values at lower values of ζ. The porous zone is dominating as ζ decreases and system gets stable for higher values of Lewis numbers $\eta\,\&\,\eta_m$.

Figure 5 depicts a variation of the R_m, and Rs_m with respect to inverse fluid and porous Lewis numbers, η & η_m. Positive increments η & η_m appear to generate more instability in the system, allowing convection to begin.

Figure 6 shows a variation of the R_m and Rs_m with respect to ζ. It is discovered that R_m increases with Rs_m regardless of the variations of ζ. We conclude that R_m has a linear relationship with R_{Sm}.

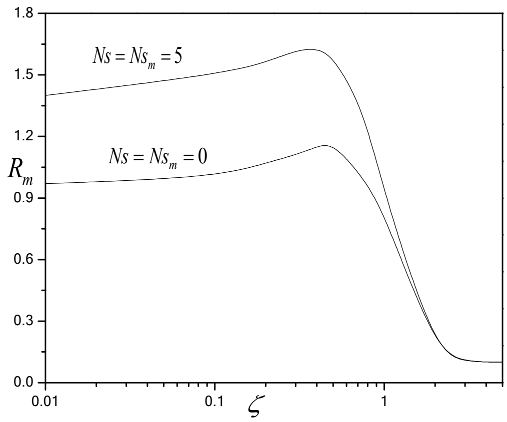

Fig. 2 R_m versus ζ for different values of N_s & N_{sm} with $\eta_m = \eta = 0.2$ & $Rs_m = 0.2$.

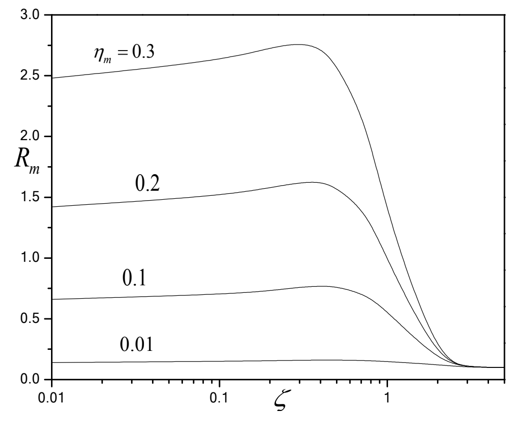

Fig. 3 R_m versus ζ for different values of η_m with $\eta = 0.2, Ns = Ns_m = 5$ & $Rs_m = 0.2$.

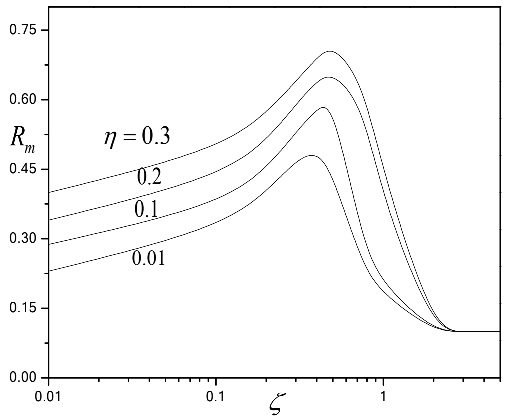

Fig. 4 R_m versus ζ for different values of η with $\eta_m = 0.2, Ns = Ns_m = 5$ & $Rs_m = 0.2$.

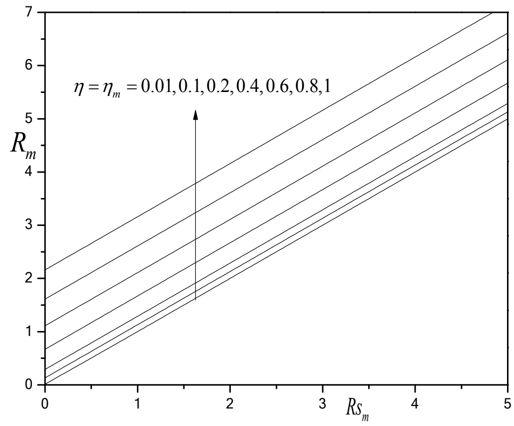

Fig. 5 R_m versus Rs_m for different values of η & η_m with $Ns = Ns_m = 5$ & $\zeta = 1$.

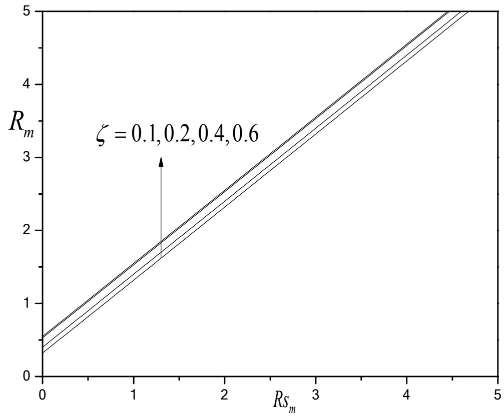

Fig. 6 R_m versus Rs_m for different values of ζ with $Ns = Ns_m = 5$ & $\eta = \eta_m = 0.2$.

6. Conclusions

Double-diffusive convective motion in a system composed of a horizontal binary fluid with a porous matrix has been investigated. The key outcomes of the study of linear stability are defined as follows:

- Stabilization of the system is achieved by increasing the internal heat sources in both fluid and porous layers.
- Depth ratio ζ increases, which means that when the fluid layer is relatively thick, the system's instability develops.
- Stabilization of the system is achieved by increasing the inverse fluid and porous Lewis numbers.
- The Rayleigh number R_m has a linear relationship with Rs_m.

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