International Journal of Thermofluid Science and Technology (2021)
Volume 8, Issue 4, Paper No. 080405
https://doi.org/10.36963/IJTST.2021080405
Research Article

A new nonlinear solution for non-Fourier heat transfer in porous fins

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ATICLE INFO

Received: 16 Apr. 2021;
Received in revised form:
14 Oct. 2021;
Accepted: 15 Oct. 2021;
Published online:
16 Oct. 2021

Keywords:
Porous fin
Non-Fourier
Non-linear
Thermal-dependent
Thermal conductivity

ABSTRACT

This article investigated nonlinear and non-Fourier heat conduction in a porous cylindrical fin since no other study has already done so. First, the literature on heat transfer in porous fins was briefly reviewed. Then, the heat conduction equation governing the problem was derived while considering all three heat conduction modes, namely conduction, convection, and radiation. The equation was made nonlinear by considering the thermal conductivity and heat generation coefficient changes caused by temperature. The equation was solved with boundary conditions and Galerkin's weighted residuals method. The comparison of results with a reference study showed that this method properly predicts the temperature profile. The effect of three parameters, namely the Vernotte number, the thermal conductivity coefficient, and the heat generation coefficient on temperature profile was investigated, which revealed the importance of assuming that the problem is non-Fourier and nonlinear.

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1. Introduction

Nowadays, competition drives manufacturing industries to reduce material consumption by as much as possible while maintaining product functionality.

In the fin manufacturing industry, the reduction in cost and dimensions must coincide with improved heat transfer, which is possible with the following three methods: 1. Increasing the fin's thermal conductivity, 2. Increasing the convection heat transfer coefficient, and 3. Increasing the heat transfer area in a specific volume.

In recent years, many studies have used the first two methods to optimize fin shape and improve heat transfer, yet there are problems associated with these two methods. First, using materials with a high thermal conductivity often increases the price. Second, there are limitations to increasing convection heat transfer coefficient, and using very high speeds is infeasible in most applications. Therefore, porous materials can be used with the third method to improve fin heat transfer [1-6].

Due to empty spaces in their solid structure, porous materials transfer heat with fluid convection which, in addition to heat conduction, shows the importance of heat transfer in porous materials.

The literature on the heat transfer of porous materials has expanded to explore various aspects such as heat convection, the effect of

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Nomenclature

A	Surface area [m^2]	*Greek symbols*	
Bi	Biot number	α_0	Thermal diffusivity [m^2 s^{-1}]
C_0	Speed of thermal wave [m s^{-1}]	β	Coefficient of volumetric thermal expansion [K^{-1}]
C_p	Specific heat [J kg^{-1}K^{-1}]	ε	Emissivity of porous fin
C_T	Dimensionless fluid temperature	λ	Coefficient of thermal conductivity [K^{-1}]
g	Gravity acceleration [m s^{-2}]	ν	Kinematic viscosity [m^2 s^{-1}]
G	Dimensionless radiation parameter	ρ	Density [kg m^{-3}]
h	Heat transfer coefficient [W m^{-2} K^{-1}]	σ	Stephen–Boltzmann constant [W m^{-2} K^{-4}]
k	Thermal conductivity [W m^{-1} K^{-1}]	τ	Thermal relaxation time [s]
K	Permeability of porous fin	ψ	Coefficient of heat generation [K^{-1}]
L	Length of the fin [m]	*Subscripts*	
$\dot{m}$	Mass flow rate of the fluid [kg s^{-1}]	a	Solid properties
P	Perimeter of the fin [m]	b	Base of fin conditions
Q	Dimensionless heat generation parameter	C	Conduction
q	Heat flux [W m^{-2}]	eff	Porous properties
S_h	Dimensionless porous parameter	f	Fluid properties
T	Temperature [K]	g	Heat generation
t	Time [s]	R	Radiation
V	Velocity [m s^{-1}]	∞	Fluid medium condition
Ve	Vernotte number	*Superscripts*	
x	Axial coordinate [m]	$\sim$	Dimensionless parameter

heat radiation, viscous dissipation, mass transfer, the effects of dimensionless numbers, the permeability effect, and others [7]. Table 1 briefly mentions the literature on the heat transfer of porous fins.

Table1
The literature on heat transfer in porous fins

Reference, year	Geometry of fin	Case study	Methodology	Main finding
[8], 2001	rectangular	Introducing a novel method to analyze porous fins	numerical	The performance of porous fins is enhanced by increasing the Rayleigh number.
[9], 2007	rectangular	Analyzing a porous fin in a convection environment	numerical	A new parameter called S_H, which includes flow and geometric parameters, was introduced.
[10], 2007	rectangular	Investigating the effect of radiation heat loss on heat transfer of porous fins	numerical	The effect of radiation is more important in cases where natural convection is weaker.
[11], 2010	rectangular	Studying the effect of MHD on the performance of porous fins.	numerical	The MHD has a conflicting behavior in porous fins, depending on temperature difference with fin base.

[12], 2011	rectangular	Considering the effects of radiation and convection heat transfer	numerical	More heat dissipates with radiation compared with convection only.
[13], 2011	rectangular	Using an analytical method to study the performance and optimization of porous fins	analytical	Different effects of parameters on the performance of porous fins were achieved.
[14], 2012	rectangular, convex, parabolic, exponential	Using an analytical method (ADM) to study different fin shapes	analytical	The fin with exponential shape transferred the most heat compared to others.
[15], 2013	rectangular	Using three analytical methods (DTM, CM, LSM) and temperature-dependent internal heat generation	analytical	The three analytical methods were found more effective than numerical methods.
[16], 2013	rectangular	Studying the effect of radiation on heat transfer of porous fins with different tip boundary conditions.	analytical	Increasing fin permeability enhances heat transfer in fins. HAM is an efficient method to analyze heat transfer in porous fins.
[17], 2013	rectangular	Solving an inverse problem to predict the values of fin permeability, thickness solid thermal conductivity, porosity, and length. .	numerical	A suitable combination of the five mentioned parameters with an error of 11 % was achieved to predict the temperature profile.
[18], 2014	rectangular, convex, triangular, exponential	Using different ceramic materials and different fin shapes	analytical, numerical	The fin with exponential shape and Si_3N_4 material transferred the most heat compared to others.
[19], 2014	triangular	Investigating the convection-radiation effects on triangular porous fin by DTM method	analytical	DTM is an excellent method to analyze similar problems.
[20], 2014	rectangular	Studying heat transfer in moving porous fins by ADM.	analytical	The temperature prediction of ADM is excellent compared to numerical methods.
[21], 2014	cylindrical	Using a hybrid method to solve an inverse problem to estimate the unknown parameters	numerical	The hybrid (DE–NLP) method is more efficient than the individual methods.
[22], 2014	rectangular	Investigating the effect of transient heat transfer on thermal response of porous fins.	numerical	The effect of different parameters on unsteady heat transfer was studied.
[23], 2015	rectangular	Applying an analytical method (HPM) to study porous fins	analytical	HPM is a powerful tool to analyze heat transfer in porous fins.
[24], 2016	annular stepped fin	Analyzing an annular stepped fin with moving conditions	analytical	A modified Peclet number was introduced to express the fin efficiency properly.

[25], 2016	rectangular stepped fin	Using an analytical method (ADM) to study porous fins with temperature-dependent parameters	analytical	ADM is a useful method to study the mentioned problem.
[26], 2016	rectangular	Solving a nonlinear radiative-conductive heat transfer problem in porous fins using SCM.	analytical	Spectral collocation method (SCM) has excellent prediction of temperature profile compared to numerical methods.
[27], 2017	rectangular	Investigating the effects of non-Fourier heat conduction and periodic thermal condition	numerical	With periodic thermal condition, non-Fourier effect was found to be important only for a short while after the initial time.
[28], 2017	trapezoidal, convex, concave	Analyzing the nonlinear problem of moving porous fins with different profiles.	analytical	Spectral element method (SEM) is a convenient method to solve nonlinear moving porous fins.
[29], 2018	rectangular, trapezoidal, concave	Using LSM to solve nonlinear heat transfer problem in porous fins	analytical	LSM is an accurate method to solve the nonlinear problem of porous fins.
[30], 2019	rectangular	Applying three analytical methods (HAM, HPM, and CM) to analyze nonlinear heat transfer in porous fins.	analytical	The effect of different parameters such as the Rayleigh number, porosity, radiation and convection parameters was found to be effective in the mentioned problem.

Due to a nonhomogeneous structure with a high degree of inhomogeneity, Fourier's law of heat conduction does not apply to porous materials in most cases, including high thermal fluxes [31]. Some references have investigated the phenomenon of non-Fourier heat transfer [32, 33].

Various experimental studies have investigated the heat transfer of porous materials in high thermal fluxes, and all have stressed the non-Fourier heat conduction model's accuracy relative to the Fourier model in the stated conditions [34-36].

Table 1 shows that studies have scarcely investigated non-Fourier heat conduction in porous fins. Only Shah Ahmadi et al. [27] have used the numerical method to investigate a linear non-Fourier heat conduction problem in a porous fin without considering radiation with the periodic boundary condition.

This study investigated non-Fourier heat conduction in a porous fin and obtained a nonlinear equation by accounting for changes in thermal conductivity coefficient and heat generation coefficient due to temperature.

2. Formulation of the problem

Fig. 1 shows a schematic of a porous cylindrical fin.

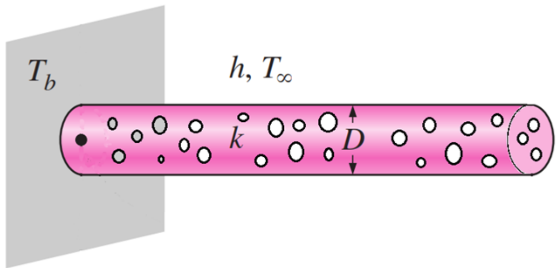

Fig. 1. Schematic of a porous cylindrical fin

According to Fig. 1, the following assumptions are considered to analyze the problem:

1- The fin is exposed to a fluid with temperature T_∞ and exchanges heat with the fluid in the modes of convection and radiation.

2- The end of the fin is insulated and the fluid flows through the fin due to its perforated structure.

3. The porous medium is assumed to be homogeneous.

4- The fin is considered one-dimensional and the heat conduction is transmitted only in the direction of its length.

5- Darcy model is used to model the interactions of the structure and the fluid, which are in thermal equilibrium.

6. Heat generation term and thermal conductivity are assumed to be a function of temperature.

Considering the above assumptions, the energy equation for the fin is written as follows:

$$\begin{aligned}&\rho_{eff}Cp_{eff}\frac{\partial T}{\partial t}+\frac{\partial q_C}{\partial x}+\frac{hP}{A}\left(T-T_\infty\right)\\&-q_g+\frac{1}{A}\frac{\partial q_R}{\partial x}+\dot{m}Cp_f\frac{\left(T-T_\infty\right)}{A\Delta x}=0\end{aligned}\tag{1}$$

The heat conduction equation is written based on the Cattaneo-Vernotte heat wave model [29, 30]:

$$\tau\frac{\partial q_C}{\partial t}+q_C+k_a\frac{\partial T}{\partial x}=0\tag{2}$$

The boundary and initial conditions for the problem are as follows:

$$\begin{aligned}&T\left(x,0\right)=0, && q\left(x,0\right)=0,\\&T\left(0,\mathrm{t}\right)=1, && q\left(\mathrm{L},\mathrm{t}\right)=0.\end{aligned}\tag{3}$$

In order to expand the equations and make them dimensionless, the following parameters are introduced:

$$\begin{aligned}&\dot{m}=\rho_f V_\infty P\Delta x, V_\infty=\frac{gK\beta}{\nu}\left(T-T_\infty\right),\\&q_R=\sigma\varepsilon(\mathrm{T}^4-T_\infty^4)P\Delta x,\tilde{\lambda}=\lambda\left(T_b-T_\infty\right),\\&\mathrm{q}_g=q_{g,a}\left[1+\psi\left(T-T_\infty\right)\right],Ve^2=\frac{\alpha_0\tau}{L^2},\\&k_a=k_{a,eff}\left[1+\lambda\left(T-T_\infty\right)\right],C_0^2=\frac{\alpha_0}{\tau},\\&\tilde{\psi}=\psi\left(T_b-T_\infty\right),Bi=\frac{PhL^2}{k_{a,eff}A},\\&\tilde{x}=\frac{x}{L},\tilde{t}=\frac{\alpha_0 t}{L^2},\mathrm{Q}_a=\frac{q_{g,a}L^2}{\left(T_b-T_\infty\right)k_{a,eff}},\\&\tilde{q}(\tilde{x},\tilde{t})=\frac{\alpha_0 q_C\left(x,t\right)}{\left(T_b-T_\infty\right)k_{a,eff}C_0},\end{aligned}\tag{4a}$$

$$\begin{aligned}&\tilde{T}(\tilde{x},\tilde{t})=\frac{T\left(x,t\right)-T_\infty}{T_b-T_\infty},\alpha_0=\frac{k_{a,eff}}{\rho_{eff}Cp_{eff}},\\&G=\frac{\sigma\varepsilon PL^2}{k_{a,eff}A}\left(T_b-T_\infty\right)^3,C_T=\frac{T_\infty}{T_b-T_\infty},\\&S_h=\frac{\rho_f gK\beta PCp_f L^2\left(T_b-T_\infty\right)}{\upsilon k_{a,eff}A}.\end{aligned}\tag{4b}$$

Using the parameters in equation (4), and differentiating equation (2) with respect to x, and differentiating equation (1) with respect to t, and performing several mathematical operations, the dimensionless equation governing heat transfer in the porous fin is as follows:

$$\begin{aligned}&\frac{\partial^2\tilde{T}}{\partial\tilde{t}^2}+\left[\frac{1}{Ve^2}+Bi-Q_a\tilde{\psi}+4G\left(\tilde{T}+C_T\right)^3\right.\\&\left.+2S_h\tilde{T}\right]\frac{\partial\tilde{T}}{\partial\tilde{t}}+\frac{1}{Ve^2}Bi\tilde{T}-\frac{Q_a}{Ve^2}\left(1+\tilde{\psi}\tilde{T}\right)\\&+\frac{G}{Ve^2}\left[\left(\tilde{T}+C_T\right)^4-C_T^4\right]\\&+\frac{S_h}{Ve^2}\tilde{T}^2-\frac{1}{Ve^2}\left(1+\tilde{\lambda}\tilde{T}\right)\frac{\partial^2\tilde{T}}{\partial\tilde{x}^2}=0.\end{aligned}\tag{5}$$

The dimensionless boundary and the initial conditions are as follows:

$$\begin{aligned}&\tilde{T}\left(\tilde{x},0\right)=0, && \frac{\partial\tilde{T}}{\partial\tilde{t}}\left(\tilde{x},0\right)=0,\\&\tilde{T}\left(0,\tilde{\mathrm{t}}\right)=1, && \frac{\partial\tilde{T}}{\partial\tilde{x}}\left(1,\tilde{\mathrm{t}}\right)=0.\end{aligned}\tag{6}$$

3. Solution procedure

Equation (5) is nonlinear due to assuming temperature-dependent thermophysical properties. It is difficult to drive the exact solution and the approximate techniques can simplify the solution procedures. In this paper, the Galerkin weighted residuals method, which is a powerful and simple method to solve nonlinear differential equations compared to other semi-analytical methods, was employed. This method solves the equations directly and does not need any small parameter or perturbation. In the current study, the Galerkin weighted residuals method was used to obtain semi-analytical solutions for the non-Fourier heat equation through the porous fins.

The Galerkin weighted residuals method is a meshless and straightforward method, which can reduce the partial differential equations to ordinary differential equations. The approximate temperature profiles (7) satisfies the essential boundary conditions (6) irrespective of the values of the unknown coefficients. As semi-analytic approximate temperature profile is employed for fin temperature, there is not any need to perform discretization in space, which simplifies the solving procedure.

To solve equation (5) by the Galerkin weighted residuals method, the following approximate profiles for the temperature are assumed:

$$\tilde{T}(\tilde{x},\tilde{t}) = 1 + \sum_{i=1}^{n} C_i(\tilde{t})\left(\tilde{x} - \frac{\tilde{x}^{i+1}}{i+1}\right) \quad (7)$$

Which satisfies the boundary conditions in (6), and $C_i(\tilde{t})$ are the unknown coefficients. Substituting the temperature profile (7) into equation (5) yields an error function R, which includes the unknown parameters $C_i(\tilde{t})$.

$$\begin{aligned} R = {} & \frac{\partial^2 \tilde{T}}{\partial \tilde{t}^2} + \left[\frac{1}{Ve^2} + Bi - Q_a\tilde{\psi} \right. \\ & \left. + 4G\left(\tilde{T} + C_T\right)^3 + 2S_h\tilde{T}\right]\frac{\partial \tilde{T}}{\partial \tilde{t}} \\ & + \frac{1}{Ve^2}Bi^2\tilde{T} - \frac{Q_a}{Ve^2}\left(1 + \tilde{\psi}\tilde{T}\right) \\ & + \frac{G}{Ve^2}\left[\left(\tilde{T} + C_T\right)^4 - C_T^4\right] + \frac{S_h}{Ve^2}\tilde{T}^2 \\ & - \frac{1}{Ve^2}\left(1 + \tilde{\lambda}\tilde{T}\right)\frac{\partial^2 \tilde{T}}{\partial \tilde{x}^2}. \end{aligned} \quad (8)$$

To obtain the values of the coefficients $C_i(\tilde{t})$, the weighted integral of the error function is equated to zero.

$$\int_0^L RW_i\left(\tilde{x}\right)d\tilde{x} = 0, \quad i = 1\left(1\right)n \quad (9)$$

The weighting functions used in equation (9) are:

$$W_i(\tilde{x}) = \left(\tilde{x} - \frac{\tilde{x}^{i+1}}{i+1}\right) \quad (10)$$

Solving equation (9) by introducing the weighting functions in (10) yields n system of second-order ordinary differential equations for the unknown coefficients $C_i(\tilde{t})$, (i=1..n). To solve this system of n second-order ODE's for the unknown coefficients, 2n initial conditions, namely $C_i(0)=0$, $\frac{d}{dt}C_i(0)=0$, (i=1..n), are required. The Galerkin weighted residuals technique is applied to the following error function of the initial condition (7):

$$\begin{aligned} & R_0 = 1 - \tilde{T}(\tilde{x},0) = \\ & 1 - \left(1 + \sum_{i=1}^{n} C_i(0)\left(\tilde{x} - \frac{\tilde{x}^{i+1}}{i+1}\right)\right) \end{aligned} \quad (11)$$

$$\begin{aligned} & \dot{R}_0 = -\frac{d}{d\tilde{t}}\tilde{T}(\tilde{x},0) = \\ & -\left(\sum_{i=1}^{n} \frac{d}{d\tilde{t}}C_i(0)\left(\tilde{x} - \frac{\tilde{x}^{i+1}}{i+1}\right)\right) \end{aligned} \quad (12)$$

Which yields:

$$\begin{aligned} & \int_0^L R_0 W_i(\tilde{x})d\tilde{x} = \int_0^L \left(1 - \tilde{T}(\tilde{x},0)\right)d\tilde{x} = \\ & \int_0^L W_i(\tilde{x})d\tilde{x} \\ & -\int_0^L 1 + \sum_{i=1}^{n} C_i(\tilde{t})\left(\tilde{x} - \frac{\tilde{x}^{i+1}}{i+1}\right)W_i(\tilde{x})d\tilde{x} \end{aligned} \quad (13)$$

$$\begin{aligned} & \int_0^L \dot{R}_0 W_i(\tilde{x})d\tilde{x} = \int_0^L \left(-\tilde{T}(\tilde{x},0)\right)d\tilde{x} = \\ & -\int_0^L 1 + \sum_{i=1}^{n} \frac{d}{dt}C_i(\tilde{t})\left(\tilde{x} - \frac{\tilde{x}^{i+1}}{i+1}\right)W_i(\tilde{x})d\tilde{x} \end{aligned} \quad (14)$$

The results obtained from equations (13) and (14) for n=3 are:

$$\begin{aligned} & C_1(0) = -\frac{1407}{22}, C_2(0) = \frac{1218}{11}, \\ & C_3(0) = -\frac{630}{11} \end{aligned} \quad (15)$$

$$\frac{d}{d\tilde{t}}C_i(0) = 0, i = 1(1)3 \quad (16)$$

In order to solve the set of ordinary differential equations in (9) with the initial conditions (15, 16) for n=3, the fourth-order Runge-Kutta method is employed. The Maple software is used for all computations.

4. Results and discussion

To verify the solution method, Fig. 2 compares the present study's results with the reference in [27]. Fig. 2 shows an excellent consistency between both results and, quantitatively, there is an average of 0.8% relative error compared to reference [27].

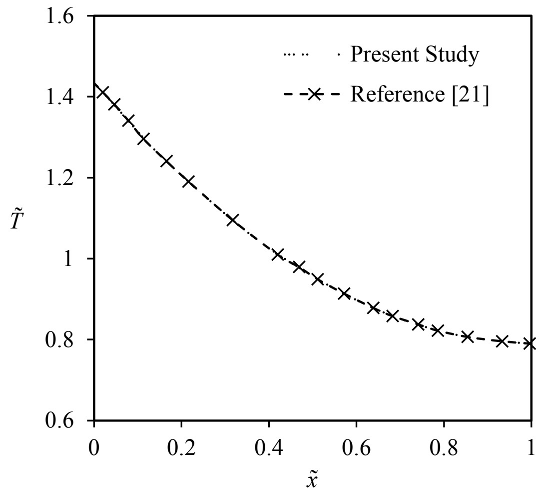

Fig. 2. Validation of solution (Ve=0, Bi=0.49, Q_a=10, G=1, C_T=1, S_h=1, $\tilde{\lambda}$=0, $\tilde{\psi}$=0)

As pointed out in the Introduction section, many studies have investigated heat transfer in porous fins and the effect of certain parameters on temperature profiles has been investigated in these studies. The effect of the Biot number in references [12, 14, 16, 20, 22, 37], the effect of changes in the C_T parameter in references [12, 16, 22, 26, 28, 38], the effect of changes in the G parameter in references [10, 12, 16, 19, 20, 22, 26, 28-30], the effect of changes in the Q parameter in references [24, 26, 28] have been studied. Moreover, the effect of changes in the S_h parameter (non-dimensional porosity) has been studied in most references given in Table1. This paper did not evaluate the effects of the mentioned parameters on temperature profiles since they were the same in Fourier and non-Fourier conditions. However, according to equation 5, the effects of these parameters on the temperature distribution inside the fin can be interpreted. Table 2 shows the effect of mentioned parameters on temperature profile.

Table 2
The effect of increasing of different parameters on temperature profile

Parameter	Effect
Bi	Decrease of temperature
C_T	Decrease of temperature
G	Decrease of temperature
Q_a	Increase of temperature
S_h	Decrease of temperature

4.1 Analyzing changes of the Vernotte number

The Vernotte number represents non-Fourier heat conduction by definition, and a higher Vernotte number represents a higher distance to Fourier heat conduction model. Fig. 3 shows changes in dimensionless temperature with four different Vernotte numbers. Fig. (3-a) shows that the temperature profile throughout the fin does not exceed zero for the Vernotte numbers greater than the Fourier number, which is the main difference between Fourier and non-Fourier heat conduction models, which proves limited thermal wave velocity in non-Fourier heat conduction model.

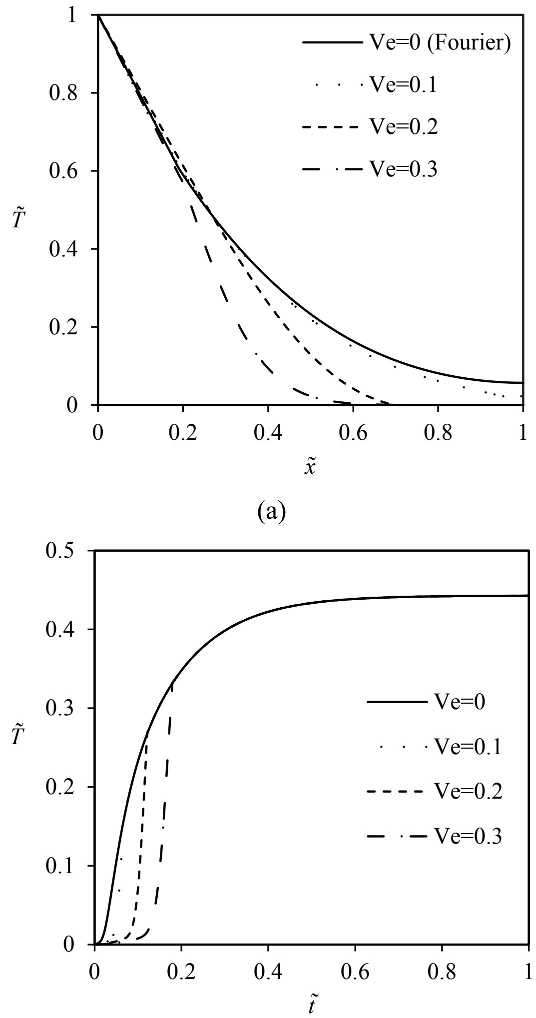

Fig. 3. The effect of the Vernotte number on dimensionless temperature profiles (Bi=0.49, Q_a=0.20, G=0.4, C_T=1, S_h=1, $\tilde{\lambda}$=0.1, $\tilde{\psi}$=0.1)

a. Over dimensionless length ($\tilde{t}$=Fo=0.1)
b. Over dimensionless time ($\tilde{x}$=0.5)

Furthermore, for $\tilde{x}$ >0.2, the temperature differences for the different Vernotte numbers are more considerable. Moreover, in higher Vernotte numbers, the temperature changes slowly for the regions with $\tilde{x}$ >0.2. The reason is that by increasing the Vernotte number and consequently, increasing the non-Fourier nature of the heat transfer, the heat wave needs more time to reach the regions that are farther from the base of the fin.

Fig. (3-b) shows the temperature profiles at $\tilde{x}$=0.5 for different values of the Vernotte numbers. It can be seen that in higher Vernotte numbers, the thermal signal reaches $\tilde{x}$=0.5, needs more time period. Although increasing the Vernotte number delays the formation of the thermal wave, but the non-Fourier nature of the heat propagation diminishes gradually after a specific period of time, the graphs become flat and the temperature is stabilized.

4.2 Analyzing changes in the dimensionless thermal conductivity coefficient

Fig. 4 shows the effect of changes in the dimensionless thermal conductivity coefficient on temperature profiles. Fig. (4-a) shows that changes in the dimensionless thermal conductivity coefficient are more pronounced in the middle of the fin. Increasing the dimensionless thermal conductivity, the velocity of the heat signal increases and the heat propagates faster through the fin. Since the temperature at the base of the fin is constant and the heat has not yet reached the ends in initial times ($\tilde{t}$=Fo=0.1), the temperature profiles of these two regions are convergent.

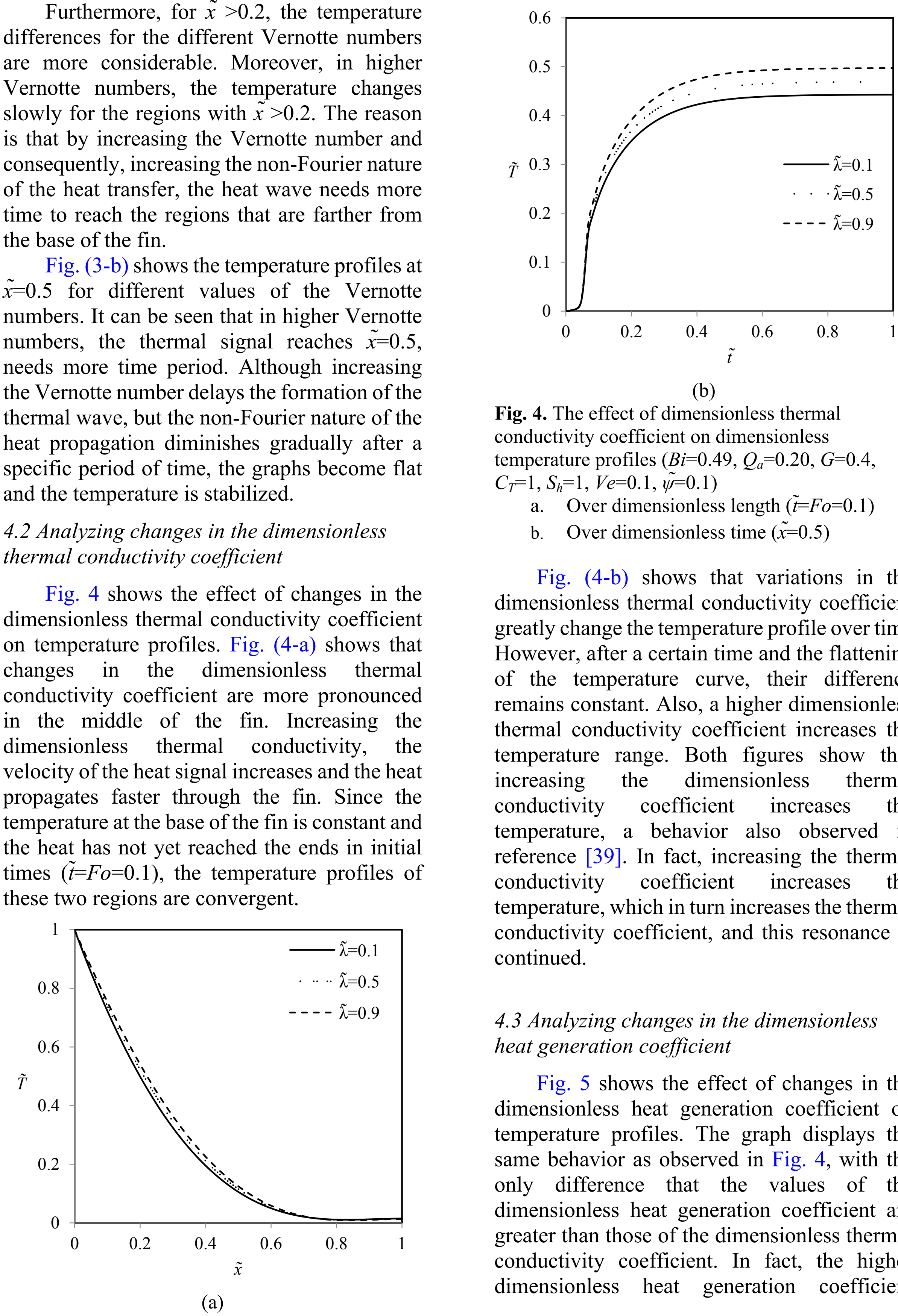

Fig. 4. The effect of dimensionless thermal conductivity coefficient on dimensionless temperature profiles (Bi=0.49, Q_a=0.20, G=0.4, C_T=1, S_h=1, Ve=0.1, ψ=0.1)

a. Over dimensionless length ($\tilde{t}$=Fo=0.1)
b. Over dimensionless time ($\tilde{x}$=0.5)

Fig. (4-b) shows that variations in the dimensionless thermal conductivity coefficient greatly change the temperature profile over time. However, after a certain time and the flattening of the temperature curve, their difference remains constant. Also, a higher dimensionless thermal conductivity coefficient increases the temperature range. Both figures show that increasing the dimensionless thermal conductivity coefficient increases the temperature, a behavior also observed in reference [39]. In fact, increasing the thermal conductivity coefficient increases the temperature, which in turn increases the thermal conductivity coefficient, and this resonance is continued.

4.3 Analyzing changes in the dimensionless heat generation coefficient

Fig. 5 shows the effect of changes in the dimensionless heat generation coefficient on temperature profiles. The graph displays the same behavior as observed in Fig. 4, with the only difference that the values of the dimensionless heat generation coefficient are greater than those of the dimensionless thermal conductivity coefficient. In fact, the higher dimensionless heat generation coefficient increases the heat generation term value and consequently more heat is generated in the fin, which increases fin temperature.

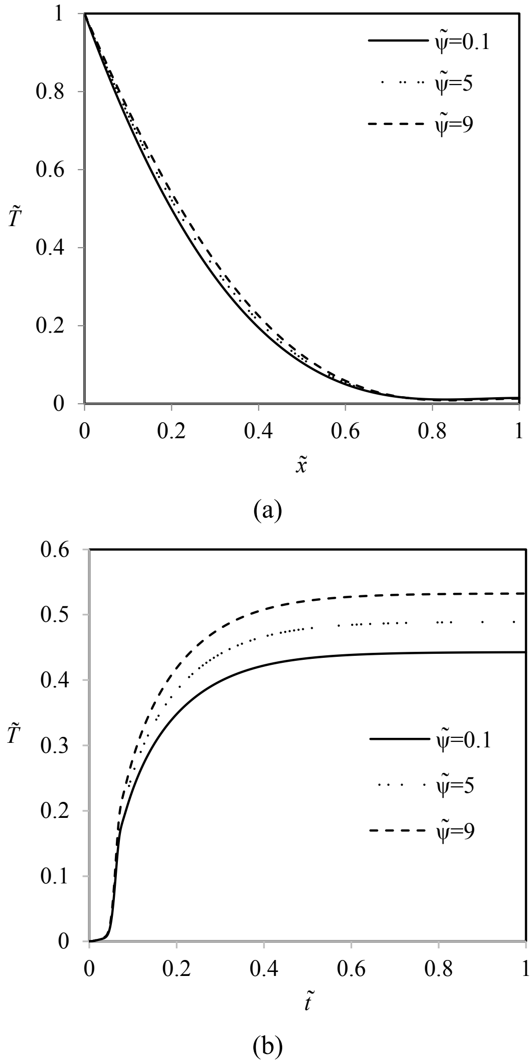

Fig. 5. The effect of dimensionless heat generation coefficient on dimensionless temperature profiles (Bi=0.49, Q_a=0.20, G=0.4, C_T=1, S_h=1, Ve=0.1, $\tilde{\lambda}$ =0.1)

a. Over dimensionless length ($\tilde{t}=Fo$=0.1)
b. Over dimensionless time ($\tilde{x}$=0.5)

5. Conclusion

This study investigated non-Fourier heat transfer in a porous fin. Since the thermal conductivity and heat generation coefficients are functions of temperature, the heat transfer equation was nonlinear, which was solved using the Galerkin's weighted residuals method. The results indicate that this method can address the problem with excellent approximation. The fin's non-Fourier heat transfer was evaluated with changes to the Vernotte number, proving that the non-Fourier assumption is important. In higher Vernotte numbers, the non-Fourier nature of the heat transfer through the fin is more noticeable. Considering non-fourier effect, a time delay occurs in the thermal signal to propagate entire the fin. Furthermore, the effect of changes in the thermal conductivity and heat generation coefficients on temperature was also investigated. Increasing the temperature-dependence of the fin thermal conductivity results in higher temperatures, especially in the middle of the fin. Moreover, in higher heat generation coefficients, more heat is generated through the fin which results in higher fin temperatures. The results showed that using materials in which these parameters change by temperature, assuming constant parameters will lead to significant errors.

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