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Research Article

Combined Impact of Vertical Throughflow and Gravity Variance on Darcy-Brinkman convection in a Porous Matrix

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ABSTRACT

The influence of the combined impact of vertical throughflow and variable downward gravity variance on a Darcy-Brinkman penetrative convective motion in a porous matrix is considered analytically. For gravity force variation, linear, quadratic, cubic, and exponential functions are considered. Stability analysis based on the small perturbation approach is performed using the assumption of normal mode. The results show that the onset of convective movement is delayed by the Petlect number, gravity variance parameter, and Darcy number, while the heat source strength parameter is improving the onset of convective movement. The scheme becomes more unstable for the cubic gravity force and more stable for the exponential downward gravity variance.

1. Introduction

The study of buoyancy-driven flows in the porous matrix is significant as it has various applications in fields such as building insulation, geothermal reservoirs, and engineering chemical reactors. The buoyancy force produced by considering internal heat generation with vertical throughflow is one of the primary sources of research interest in the onset of convective movement in a porous matrix. Many authors researched the beginning of convection in a porous matrix using the Darcy rule (Lapwood [1], Barletta et al. [2], Brevdo and Ruderman [3], Chen [4], Nield [5], Nield and Kuznetsov [6], Gangadharaiah [7] Suma et al. [8] Shivakumara et al. [9]). However, a drawback of Darcy law is that it does not contain the viscous energies practiced by the particle. In this manner, it does not permit the role of a no-slip boundary situation on the plane of the particle.

To overcome this, Brinkman [10] proposed the accumulation of an extra viscous term for the Darcy model to solve this problem, leading to the development of the Darcy-Brinkman model. Many researchers were drawn to study the impact of throughflow

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along with or without internal heat generation using the Darcy-Brinkman model as it offers the opportunity to control convective instability(Nield, [11], Rees [12], Deepika et al. [13], Yadav et al. [14], Wang and Tan [15], Yadav et al. [16] and Amit Mahajan and Reena [17]).

It is noted that in a wide variety of significant circumstances, the changeable gravity effect with the height of its planes for large-scale flows in geophysical research, engineering, atmosphere, and sea. We generally disregard this variation of gravity in laboratory experiments and conclude that the force of gravity is constant. But, to a large degree, understanding gravity as a variable quantity is important. In this way, it seems to be necessary to analyze the convective movement in a porous matrix with a variable gravity effect. The study of the impact of the variable gravity effect in a porous bed is, however, very limited. Alex and Patil [18] investigated the downward gravity effect with Darcy penetrative convection in a porous bed and found that the stability of the arrangement is improved by a reduction in the gravity factor. The effect of linear variable gravity variance and throughflow penetrative convective motion in a porous matrix was reported by Suma et al. [19] using regular perturbation technique. Gangadharaiah et al. [20] investigated the variable gravity field and throughflow effects on penetrative convection in a porous layer. Surface-driven convective motion in a composed system with binary fluid is investigated by Gangadharaiah [21]. Dhananjay Yadav [22]and Nagarathnamma et al. [23] investigated variable gravity field effects on porous layers by using Galerkin technique. The main objective of this study is to analyze the mutual effect on the beginning of Darcy-Brinkman convective movement in a porous bed with throughflow and a changeable downward gravity effect. By means of normal perturbation techniques, analytical findings are derived from the predominant equations. The outcomes of various relevant convection arrival parameters have been presented in detail.

2. Conceptual Model

Consider an infinite horizontal layer of an isotropic porous matrix of width d and uniformly distributed internal heat source Q with variable downward gravity field $g(z)$ is given by $\vec{g}(z) = -g_0\left(1+\lambda H(z)\right)\hat{k}$, is operates in the direction of negative $z-axis$. The temperatures of the lower and upper boundaries are taken to be uniform and equal to T_l and T_u respectively, with $T_l > T_u$.

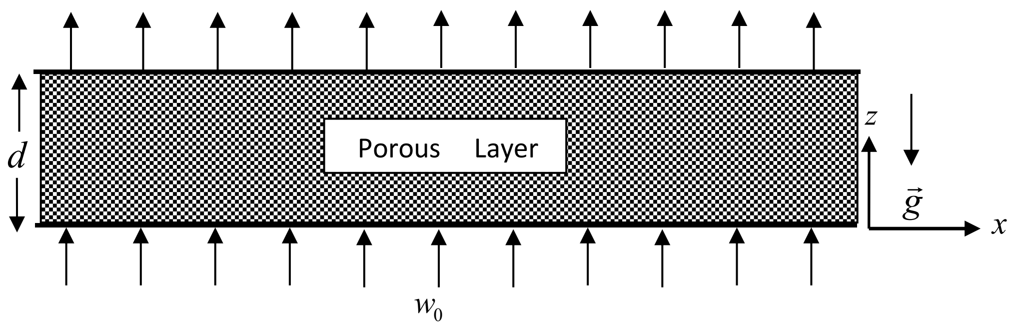


Figure 1. Physical configuration

Nomenclature

a	horizontal wave number	***Greek symbols***	
D	differential operator	ε_T	ratio of thermal diffusivities
Da	Darcy number	κ	thermal diffusivity
K_m	thermal conductivity	μ	fluid viscosity
W	perturbed vertical velocity	β_T	thermal expansion coefficients
Pr	Prandtl number	∇_h^2	horizontal Laplacian operator
Pe	Peclet number	∇^2	Laplacian operator
$\vec{V}$	velocity vector (u, v, w)	ϕ	porosity of the porous medium
d	thickness of the porous layer	θ	amplitude of perturbed temperature
λ	gravity parameter	ρ_0	fluid density
p	pressure	v	kinematic viscosity
$H(z)$	gravity variance	***Subscripts***	
T	temperature	b	basic state
Ns	dimensionless heat source strength	l	lower
Q	uniform internal heat source	u	upper

The basic governing equations are

$$\nabla \cdot \vec{V} = 0 \tag{1}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial \vec{V}}{\partial t} = -\nabla p - \frac{\mu}{K}\vec{V} + \mu_m \nabla^2 \vec{V} - \rho_0\left[1-\beta_T\left(T-T_0\right)\right]\vec{g}(z) \tag{2}$$

$$\left(\rho c\right)_m \frac{\partial T}{\partial t} + \left(\rho c\right)\left(\vec{V}\cdot\nabla\right)T = K_m \nabla^2 T + Q \tag{3}$$

Then the basic steady-state solution is

$$\left(u, v, w, p, T\right) = \left(0, 0, w_0, p_b(z), T_b(z)\right) \tag{4}$$

The basic state temperature is given by

$$T_b(z) = T_l + \frac{Q\,d}{W_0}\left[\left(\frac{z}{d} + \frac{1-e^{W_0 z/\kappa}}{e^{W_0 d/\kappa}-1}\right)\right] - \left(T_l - T_u\right)\left(\frac{1-e^{W_0 z/\kappa}}{e^{W_0 d/\kappa}-1}\right) \quad 0 \le z \le d \tag{5}$$

The impact of Throughflow is to adjust the $T_b(z)$ from linear to an exponential with regard to vertical coordinates in a porous matrix (Fig.2). Infinitesimal disruptions are superimposed in the form of exploring the stability of the basic state.

$$\vec{V} = W_0\hat{k} + \vec{V}', \; T = T_b(z) + \theta, \; p = p_b(z) + p' \tag{6}$$

Where the perturbations are the primed quantities and are considered to be small and linearized in the normal way. The expression for pressure is omitted from Eq. (2) finally, we get

$$\frac{1}{P_r}\frac{\partial}{\partial t}\left(\nabla^2 w\right) = \nabla^2 w - Da\nabla^4 w + R\nabla^2{}_H \theta\left(1+\lambda\; H(z)\right) \tag{7}$$

$$\left(A\frac{\partial}{\partial t} + Pe\frac{\partial}{\partial z} - \nabla^2\right)\theta = -w\left[\frac{Ns}{Pe} + (Pe+2Ns)\frac{Pe\; e^{Pe z}}{\left(1-e^{Pe}\right)}\right] \tag{8}$$

Where $A = \left(\rho c\right)_m / \left(\varepsilon\rho c\right)$ heat capacity ratio, $Pe = W_0 d/\kappa$ is the Peclet number, $R = \beta_T K g \nabla T d / \mu\,\alpha$ is the Rayleigh number,

$Ns = qd^2 / 2K\nabla T$ is the heat source strength, and $Da = K / d_m^2$ is the Darcy number.

We assume the solution is of the form

$$\left(w,\theta\right) = \left[W\left(z\right),\Theta\left(z\right)\right]\exp\left[i\left(lx+my\right)\right] \quad (9)$$

and substituting this in Eqs. (7) – (8), we get

$$\left(D^2-a^2\right)\left[Da\left(D^2-a^2\right)-1\right]W$$
$$= a^2 R\left(1+\lambda\, H\left(z\right)\right)\Theta \quad (10)$$

$$\left(D^2-a^2-Q\,D\right)\Theta = f\left(z\right)W \quad (11)$$

Where $f\left(z\right)=\left\{\dfrac{2Ns}{Pe}-\dfrac{\left(2Ns+Pe\right)e^{Pe z}}{\left(e^{Pe}-1\right)}\right\}$.

And

$$W = DW = D\Theta = 0 \;\; at \;\; z=0 \quad (12)$$

$$W = D^2W = D\Theta = 0 \;\; at \;\; z=1 \quad (13)$$

are boundary conditions.

3. Method of Solution

The analytical solutions of the eigenvalue problem Eq.(10) and Eq.(11) are obtained by using a using regular perturbation procedure. Accordingly, the variables W, and Θ are expanded in powers of a^2 as

$$\left(W,\Theta\right) = \sum_{i=0}^{N}\left(a^2\right)^i\left(W_i,\Theta_i\right) \quad (14)$$

Substitution of Eq. (14) into Eqs. $(10)-(11)$ and the boundary conditions $(12)-(13)$, zeroth- order equations are

$$Da\; D^4W_0 - D^2W_0 = 0 \quad (15)$$

$$D^2\Theta_0 - PeD\Theta_0 = W_0 f(z) \quad (16)$$

and

$$W_0 = 0,\; D\Theta_0 = 0,\; DW_0 = 0 \text{ at } z=0 \quad (17)$$

$$W_0 = 0, D\Theta_0 = 0, D^2W_0 = 0 \text{ at } z=1 \quad (18)$$

The solution to the zeroth-order is given by

$$W_0 = 0, \quad \Theta_0 = 1 \quad (19)$$

At the first order in a^2 Eqs. $(10)-(11)$ then reduce to

$$Da\,D^4W_1 - D^3W_1 = R\left(1+\lambda\; H\left(z\right)\right) \quad (20)$$

$$D^2\Theta_1 - Pe\,D\Theta_1 = W_1\, f(z)+1 \quad (21)$$

and the boundary conditions $(12)-(13)$ become

$$W_1 = 0, D\Theta_1 = 0, DW_1 = 0\, at \quad z=0 \quad (22)$$

$$W_1 = 0, D\Theta_1 = 0, D^2W_1 = 0 \text{ at } \quad z=1 \quad (23)$$

Equations (21) involving $D^2\Theta_1$ gives us

$$\int_0^1 f(z)W_1\,dz = -1. \quad (24)$$

The Eq. (20) together with solvability condition Eq. (24) are solved using *MATHEMATICA;* the R^c is obtained for four different cases of variable gravity functions.

4. Outcomes and discussion

In this study, four types of variations in the gravitational force are considered: linear, quadratic, cubic, and exponential. The gravity force functions are taken as $Case\left(i\right): H\left(z\right)=-z$ (for linear variations), $Case\left(ii\right)$: $H\left(z\right)=-z^2$ (for quadratic variations), $Case\left(iii\right)$: $H\left(z\right)=-z^3$ (for cubic variations), and $Case\left(iv\right)$: $H\left(z\right)=-\left(e^z-1\right)$ (for exponential variations). The influence of the throughflow parameter, gravity variation parameter, heat source parameter, and Darcy number on the R^c is calculated, and outcomes are presented in Figs. 4 to 6. To confirm our calculation accuracy, the results obtained without internal heating ($Ns=0$) and throughflow ($Pe=0$), we recover the known exact value $R^c=12$ (Nield and Bejan [24]).

Figure 3 indicates the deviance of R^c as a function of λ for different values of Peclet number Pe for four distinct cases ($Case\left(i\right),\left(ii\right),\left(iii\right)\&\left(iv\right)$) of gravity field variations. The value of the essential R^c increases on increasing Pe is recognized, so the feeling of increasing Pe delays the arrival of the convective movement. This took place because the imperative heat gradients are changed progressively to a layer of the boundary where the throughflow is specified. In addition, the stability of the convection is found to be higher for $Case(iv)$ and the instability is higher for the gravity variance function of $Case(iii)$.

Figure 4 displays the deviance of R^c as a function of λ for different values of Da for four cases ($Case(i),(ii),(iii)\&(iv)$) of gravity field variations. Both Da and λ have been found to improve the structure's stability. This occurred because the outcome of growth is to increase the viscous effects of the system. As λ is increased, the critical Rayleigh number R^c increases because of an improvement in the value of λ decreases in the frequency of gravity force. Since the disruption in the system reappears when the force of gravity decreases and the beginning of the convective wave is delayed. With an increase in the estimate Da , the size of the convective cells decreased while with λ it decreased. In addition, it is known that the scheme has more stability for $Case(iv)$.

Figures 5,6, and 8 demonstrate the effect of internal heat source variation on the system. From Figure 5, it is recognized that the mechanism stabilizes both downward and upward throughflow for distinct gravity field variations without heat source strength. In the absence of Ns , the path of the throughflow has no effect on the scheme's stability and delays the onset of convection. From Figure 6, it is noted that the for $Pe > 0$ has no influence on the stability and for $Pe < 0$ destabilizes the system when $-1.7 \le Pe \le -1.2$ with Ns. The R^c plotted against the Ns is showed in Fig. 7 for the values of $Da = 0.01$ and $Pe = 5$ for four cases ($Case(i),(ii),(iii)\&(iv)$) of gravity field variations. The value of R^c decrease can be observed as the value of Ns increases. As a consequence, the internal heat source increases the medium's global temperature and induces the start of instability.

The disturbed vertical velocity functions is shown in Figure 8 for various values of Ns for $Da = 0.01$ and $Pe = 5$. In the absence of Ns in the porous matrix is to accelerate W compared to in the presence of internal heating for all for four distinct cases of downward gravity forces.

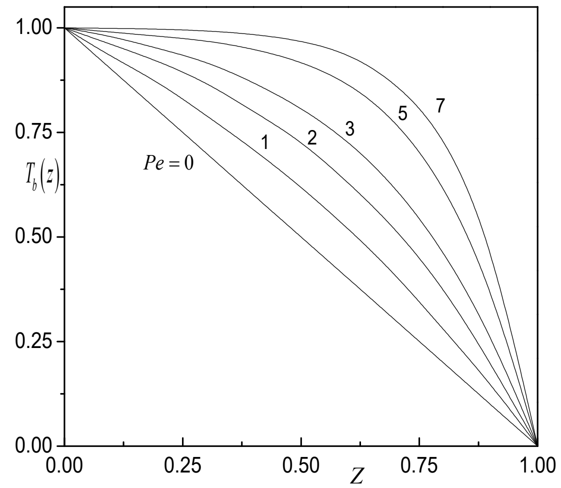
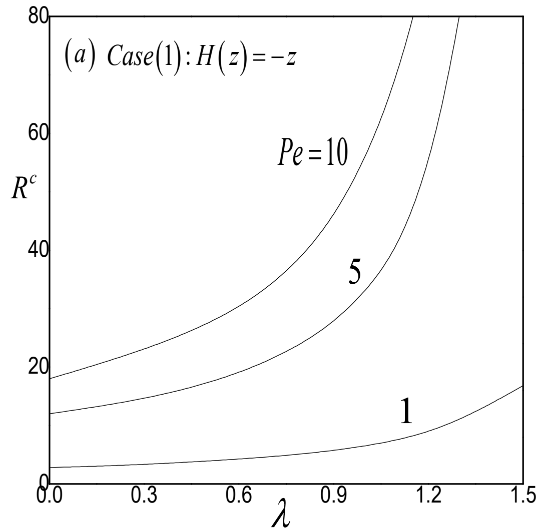


Fig. 2 Plot of the basic state temperature distributions for different values of $T_b(z)$ with $Ns = 0$.



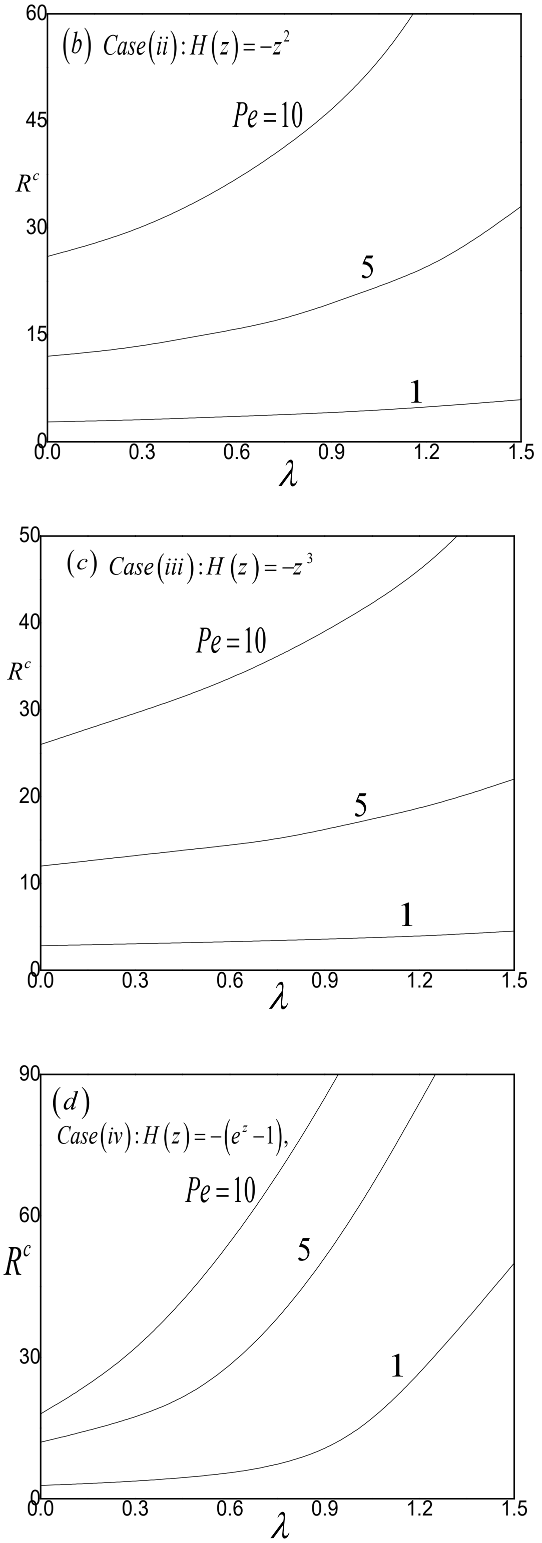
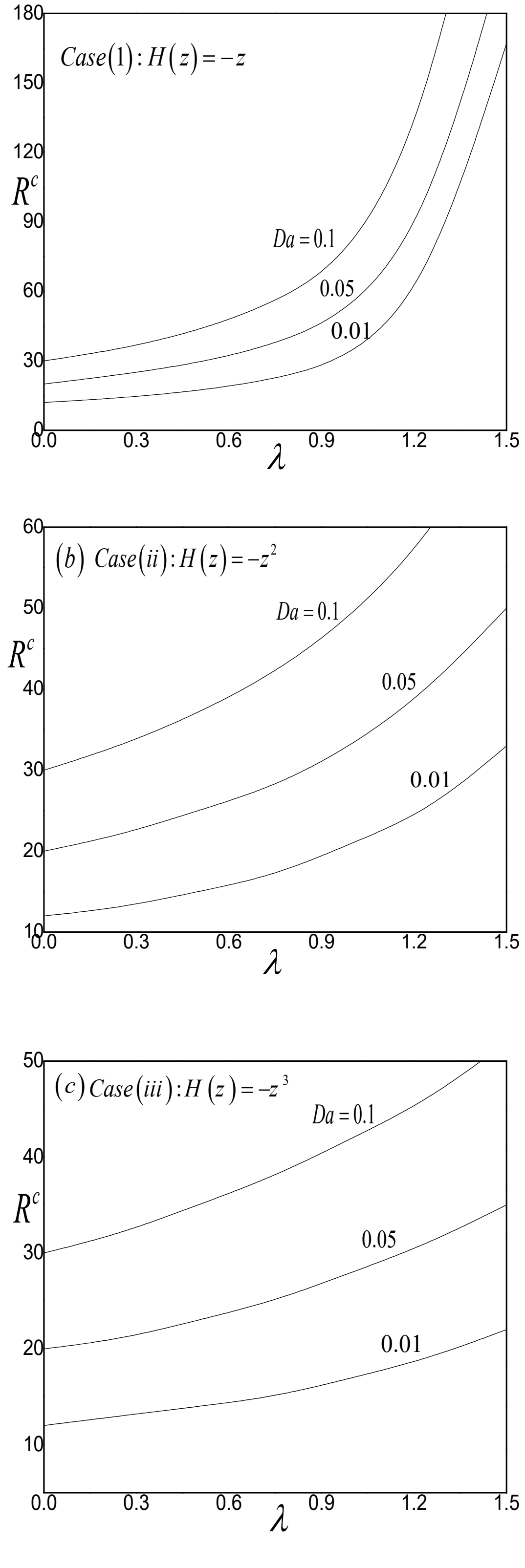


Figure 3. Variation of R^c with λ for various values of Pe at $Da = 0.01$ & $Ns = 5$ for cases $(a): H(z) = -z, (b): H(z) = -z^2,$ $(c): H(z) = -z^3,$ and $(d): H(z) = -(e^z - 1).$



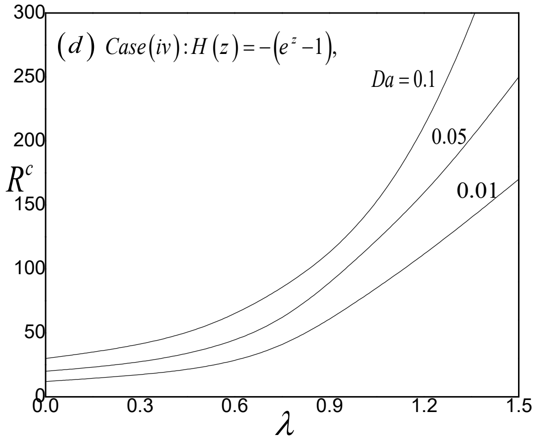


Figure 4. Variation of R^c with λ for different values of *Da* at *Pe*=5=*Ns* for cases $(a): H(z) = -z, (b): H(z) = -z^2$, $(c): H(z) = -z^3$, *and* $(d): H(z) = -(e^z - 1)$.

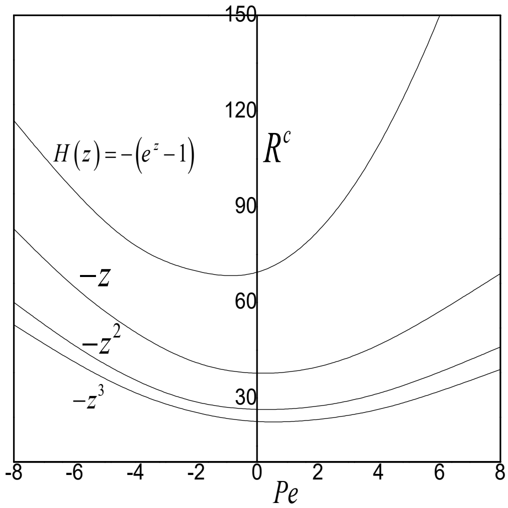


Figure5. Variation of R^c with Pe at $Ns = 0$ $\&Da = 0.01$ for all cases of gravity variance.

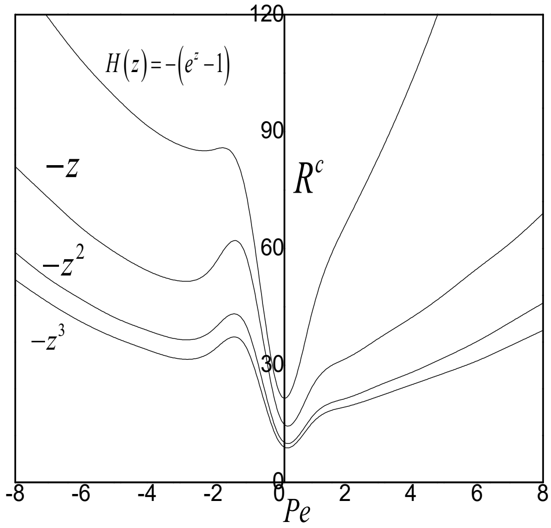


Figure6. Variation of R^c with Pe at $Ns = 5$ $\&Da = 0.01$ for all cases of gravity variance.

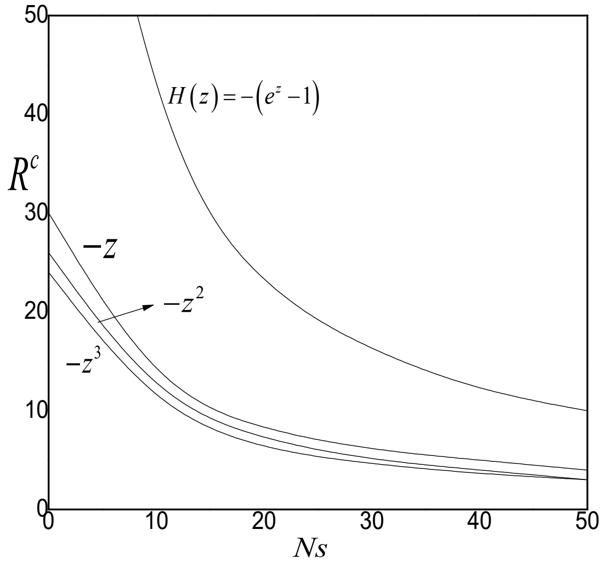


Figure7. Variation of R^c with Ns at $Pe = 5$ & $Da = 0.01$ for all cases of gravity variance.

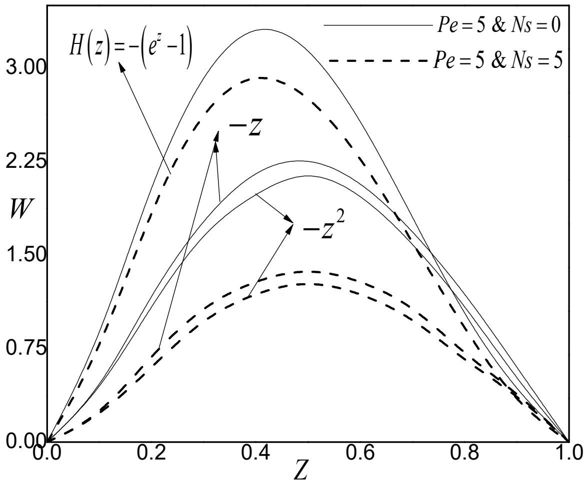


Figure8: Variation of W with Z for different values of Ns for all cases of gravity variance with $Pe = 5$.

5. Conclusions

Changeable gravity fields with height, due to the combined impact of vertical throughflow and an inner heat source, the thermal convection in a porous matrix is studied. The analysis takes into account four distinct forms of variations in the gravity field. The key outcomes of the study of linear stability are defined as follows:

- The effect of increasing Pe, λ and Da is discovered to prolong the onset of convective motion while Ns is reacted to increase the onset of convective motion.
- As the effect of the λ, Da and upward throughflow $(Pe > 0)$ increases, the size of the convective cells decreases, whereas the internal heat source parameter has a double character on the convection cell scale.
- It is distinguished that for cubic gravity force, the flow is more stable, and for exponential gravity force, the flow is more unstable.

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