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Research Article

Mixed convective peristaltic flow of an Oldroyd 4-Constant fluid in a planner channel

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Abstract

Mixed convective peristaltic flow of an Oldroyd 4-constant fluid in a two dimensional channel with flexible walls was studied in this article. The arising complicated nonlinear equations are reduced into solvable form using small Reynolds number and long wavelength assumption. The resulting equations are solved numerically by shooting method and implicit finite difference scheme. The results are compared for both velocity and temperature profiles and also compared with available literature. The impacts of involved parameter on non-dimensional velocity, temperature, pressure gradient, pressure rise per wave length and trapping are presented graphically. By the influence of buoyancy forces the symmetry of the velocity profile is disturbed about the central line of the channel and also the size of the trapped bolus increases in the left half of the channel. Also the decrease in the pressure drop per wavelength is observed for the increasing values of thermal Grashof number.

Keywords: Peristalsis, mixed convection, Oldroyd 4-constant fluid, shooting method

1. Introduction

The word peristalsis originally evolved from new Latin and is derived from the Greek word “peristallein” meaning “to wrap around” which is combination of two words peri-, “around” and stallein-, “to place”. Thus peristalsis is the type of motion due to contraction and relaxation of the boundary of the container in radially symmetrical direction. From the above description, it is evident that such type of motion occurs not only in industry but also have huge usage by nature as in many physiological processes the flow of fluids are blessed due to this mechanism. An instrument called peristaltic pump is used to pump the fluid without direct contact with it. This particular advantage of peristaltic pump made it useful in many medical instruments also. Physiologically contraction and relaxation mechanism involves in excretory system, menstrual

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cycle, chyme movement in digestive system, blood circulation in the capillaries and in many more bio fluids flow. Owing to its wide applications, for the last two decades, many researchers devoted their time to investigate about peristalsis. The mechanism by which the peristaltic motion occurs was studied by Latham [1] firstly in relation to mechanical pumping. Under negligible inertia and streamline curvature effects peristaltic flow for the Newtonian fluid case is studied by Shapiro et.al [2]. The mechanism of peristalsis in planner channel was studied by Fung and Yih [3] for arbitrary Reynolds and wave numbers. Raju and Devanathan [4] theoretically analyzed the peristaltic motion of power- law fluid in a tube. They developed the solution for stream function as a power-series in terms of amplitude of the wave. Peristaltic pumping of second order fluid in planner channel and axisymmetric tubes was analyzed by Siddiqui et al. [5] and Siddiqui and Schwarz [6], respectively. Usha and Rao [7] researched on the impact of peripheral layer in peristaltic flow of power-law fluid. Hayat et al. [8] studied the interaction of peristalsis with rheology by utilizing the constitutive equations of Johnson-Segalman and Oldroyd-B fluids. Mekheimer and Elmaboud [9] investigated the peristaltic motion using couple stress fluid. Hayat et al. [10] studied the peristaltic motion of a Burger's fluid in a planar channel. Peristaltic Creeping Flow of Power Law physiological fluids through a non uniform Channel with slip Effect has been discussed by Chaube et al. [11]. More recently Ali et al. [12] investigated the peristaltic motion in planner channel for an Ellis fluid. Peristaltic flow of a conducting Williamson viscoelastic fluid has been explored by Bég and Rashidi [13]. Tripathi and Beg [14] studied peristaltic propulsion of Burger's fluid through a non-uniform porous medium using homotopy perturbation method and variational iteration method. In another study, Tripathi and Beg [15] carried out an analytical study of transient magneto-peristaltic flow of couple stress biofluids.

Due to the temperature gradient between the contacting and relaxing walls of the fluid container and moving fluids inside, the phenomena of heat transfer takes place. More work on peristalsis has been done considering the heat transfer because in many industrial and biological applications transfer of heat happens with motion of the concerned fluid. In physiological processes such as heat conduction in tissues, in the processes of hemodialysis and oxygenation, heat convection due to the blood flow from the pores of the tissues and radiation between surface and its environment, heat transfer involves. Therefore, in literature many studies are available which describes the heat transfer with peristalsis. Flow with heat transfer for a viscous fluid in annulus has been examined by Vajravelu et al. [16] to show that peristaltic wave effects are evident on heat transfer. Tripathi [17] discussed a mathematical model for swallowing of food bolus through the esophagus under the influence of heat transfer. Tripathi and Beg [18] presented the unsteady physiological magneto-fluid flow and heat transfer through a finite length channel by peristaltic pumping. Sobh [19] studied the slip effects on peristaltic transport of a magneto-Newtonian fluid through a porous medium with heat transfer. Hayat et al. [20] studied the effect of heat transfer on the peristaltic flow of an electrically conducting fluid in a porous space.

The term convection refers to collective movement of groups or aggregates of molecules within fluids. Convection can be categorized as natural convection and forced convection. In urination the phenomena of natural convection involves, while, in many industrial and biological processes forced convection plays important role in heat transfer. The combination of both types of convection is termed as mixed convection. In fluids due to variation of gravitational body force associated with non-uniformity of density within the system, the flow field is significantly modified from that which would prevail under conditions of uniform density. In view of the above discussion to best of our information there is not any attempt available which investigate the mixed convective flow of Oldroyd 4-constant fluid in a planner channel due to peristalsis.

In this paper, we investigate mixed convection on the peristaltic flow of an Oldroyd 4-constant fluid in a planner channel. The organization of the paper is as follow. The problem is formulated in section 1. The effects of pertinent parameters on various characteristic of peristaltic motion are discussed in detail in section 2. Graphical and numerical results are compared in section 3 and some main findings are presented in section 4.

2. Formulation of the problem

Let us consider the flow of an incompressible Oldroyd 4-constant fluid in a symmetric channel of width $2a_1$. The temperature T at the left and right walls are T_0 and T_1 respectively. The shape of the wave propagated on the walls of channel is

$$\pm h(X,t) = \pm\left(a_1 + b_1 cos\left[\frac{2\pi}{\lambda}(X - c\bar{t})\right]\right), \tag{1}$$

where b_1 is the wave amplitude, λ is the wavelength and $\bar{t}$ is the time. The law of conservation of mass and momentum, and energy are

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{2}$$

$$\rho_f\left[\frac{\partial U}{\partial t} + \left(U\frac{\partial}{\partial X} + V\frac{\partial}{\partial Y}\right)U\right] + \frac{\partial P}{\partial X} = \frac{\partial S_{XX}}{\partial X} + \frac{\partial S_{XY}}{\partial Y} + \rho_f g\alpha[(T - T_0)] \tag{3}$$

$$\rho_f\left[\frac{\partial V}{\partial t} + \left(U\frac{\partial}{\partial X} + V\frac{\partial}{\partial Y}\right)V\right] + \frac{\partial P}{\partial Y} = \frac{\partial S_{XY}}{\partial X} + \frac{\partial S_{YY}}{\partial Y} \tag{4}$$

$$\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \frac{\kappa}{(\rho c_f)}\left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}\right) + \frac{1}{(\rho c_f)}\begin{pmatrix} S_{XX}\frac{\partial U}{\partial X} + \\ S_{XY}\left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X}\right) + S_{YY}\frac{\partial V}{\partial Y}\end{pmatrix} \tag{5}$$

where the extra stress tensor for Oldroyd 4-constant fluid can be expressed as

$$S + \lambda_1\frac{DS}{Dt} + \lambda_3 tr(S)A_1 = \mu\left(1 + \lambda_2\frac{D}{Dt}\right)A_1 \tag{6}$$

in which μ is the dynamic viscosity, λ_1 and λ_3 are the relaxation times parameter, λ_2 is the retardation time parameter. Here U, V, p, μ, g, k, T and axial velocity, transverse velocity, pressure, fluid viscosity, gravitational acceleration. Introducing the variables

$$x^* = \frac{2\pi x}{\lambda},\ y^* = \frac{y}{a_1},\ u^* = \frac{u}{c},\ v^* = \frac{v}{c},\ h^* = \frac{h(x)}{a_1},\ p^* = \left(\frac{2\pi a_1^2}{\lambda\,\mu\,c}\right)p,\ S^* = \left(\frac{a_1}{\mu c}\right)S$$

$$\theta = \frac{T - T_0}{T_1 - T_0}, \delta = \frac{2\pi a_1}{\lambda}, u = \frac{\partial\psi}{\partial y^*},\ v = -\delta\frac{\partial\psi}{\partial x^*},\ \lambda_1^* = \frac{a_1}{c}\lambda_1,\ \lambda_2^* = \frac{a_1}{c}\lambda_2,\quad \lambda_3^* = \frac{a_1}{c}\lambda_3$$

The governing equations in wave frame after using $\delta \ll 1$ and low Reynolds approximation

$$\frac{\partial p}{\partial x^*} = \frac{\partial S_{xy}^*}{\partial y} + G_r\theta \tag{7}$$

$$\frac{\partial p}{\partial y^*} = 0 \tag{8}$$

$$\left(\frac{\partial^2\theta}{\partial {y^*}^2}\right) + Br\left(S_{xy}^*\left(\frac{\partial^2\psi}{\partial {y^*}^2}\right)\right) = 0 \tag{9}$$

where

$$S_{xy}^* = \frac{1+2\alpha_1\left(\frac{\partial^2\psi}{\partial {y^*}^2}\right)^2}{1+2\alpha_2\left(\frac{\partial^2\psi}{\partial {y^*}^2}\right)^2}\left(\frac{\partial^2\psi}{\partial {y^*}^2}\right) \tag{10}$$

where $Re = \frac{\rho c a_1}{\mu}, \frac{\kappa}{\mu c_f} = \frac{1}{Pr}, Ec = \frac{c^2}{(c_f)\,(T_1-T_0)}, Br = EcPr$ and $Gr = \frac{(T_1-T_0)g\alpha\, a_1^2}{c\,\nu}$ Reynolds Number, dimensionless temperature, Prandtle , Eckert , Brinkman number and Grashof number. Dropping prime and eliminating pressure from (7), we can write

$$\frac{\partial}{\partial y^2}\left(\frac{1+2\alpha_1\left(\frac{\partial^2\psi}{\partial y^2}\right)^2}{1+2\alpha_2\left(\frac{\partial^2\psi}{\partial y^2}\right)^2}\frac{\partial^2\psi}{\partial y^2}\right) + Gr\frac{\partial\theta}{\partial y} = 0 \tag{11}$$

$$\left(\frac{\partial^2\theta}{\partial y^2}\right) + Br\left(\frac{1+2\alpha_1\left(\frac{\partial^2\psi}{\partial y^2}\right)^2}{1+2\alpha_2\left(\frac{\partial^2\psi}{\partial y^2}\right)^2} * \left(\frac{\partial^2\psi}{\partial y^2}\right)^2\right) = 0 \tag{12}$$

The dimensionless boundary conditions in the wave frame are given by

$$\psi = \pm\frac{F}{2}, \quad \frac{d\psi}{dy} = -1\,, \quad \theta = \begin{Bmatrix}1\\0\end{Bmatrix} \quad \text{at} \quad \pm\eta = \pm 1 + \phi Cos(x) \tag{13}$$

Dimensionless pressure rise is given by

$$\Delta p = \int_0^{2\pi}\frac{dp}{dx}dx, \tag{14}$$

Θ is Dimensionless mean flow in laboratory frame where F is in wave frame can be related as,

$$\Theta = \mathrm{F} + 2, \tag{15}$$

The heat flux at upper wall of the channel is

$$Z = \frac{\partial h}{\partial x}\frac{\partial\theta}{\partial\eta}\bigg|_{y=h} \tag{16}$$

3. Results and discussions

In this section we examine the effects of thermal Grashof number (Gr), for flow characteristics of inquired peristaltic motions. The effects of Gr on velocity profile are represented in Fig. 2(a), the symmetry of the curves is disturbed as we increase the thermal Grashof number. For $Gr = 0$ no gravity effects and for $Gr = 1$ both gravity and viscous effects are examined. As we increase the values of Gr the gravity effects are increasing. The effects of thermal Grashof number on stream function ψ are presented in Fig 2(b). It is obvious fact that as we increase the thermal Grashof number the stream function shows a valuable decrease.

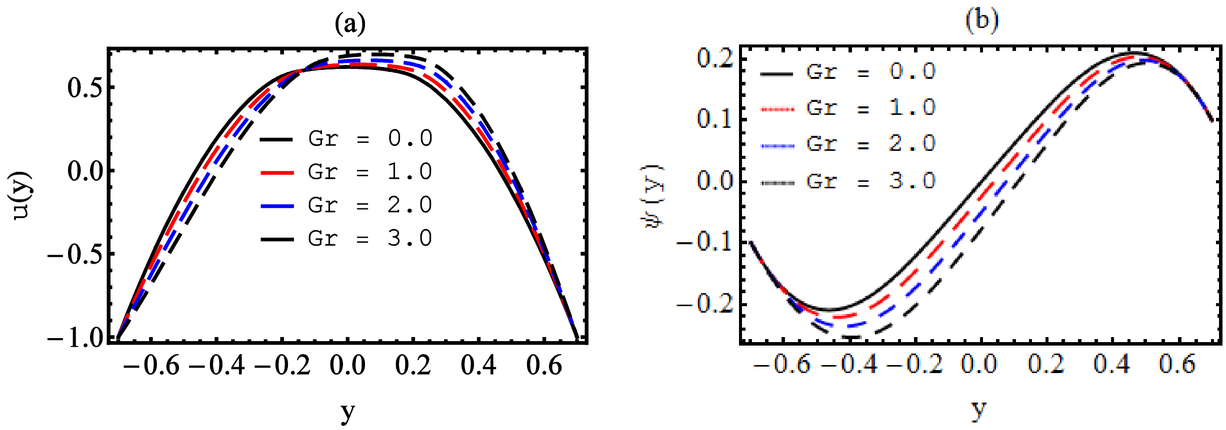


Fig.2 Effects of Gr on (a) Velocity profile and (b) stream function when $\varphi = 0.3,\ F = -0.2,\ \alpha_1 = 0.5,\ \alpha_2 = 2.0,\ Br = 2.5$

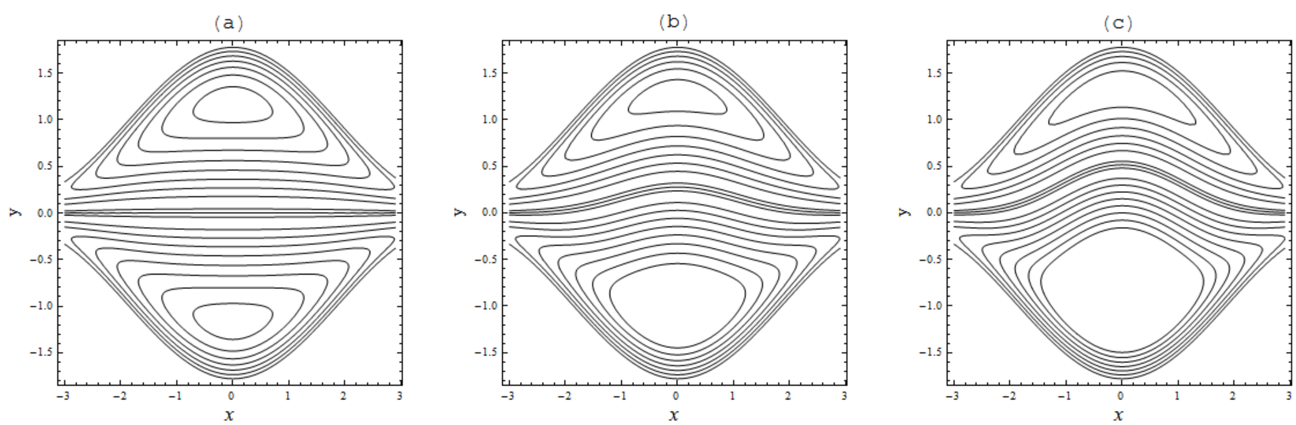


Fig.3 Stream lines for (a) *Gr=0.0,* (b) *Gr=0.5,* (c) *Gr=1.0* when $\varphi = 0.8,\ F = -0.2,\ \alpha_1 = 0.5,\ \alpha_2 = 1.5,\ Br = 1.0$

The effect of thermal Grashof number is represented in Fig. 3. Fig. 3(a) is for Gr = 0 which show that viscous forces are dominant and symmetry is the due to the no influence of gravity force. As we increase the gravity effects the symmetry of the stream lines is disturbed and the disturbance in the flow is transferred to the upper half of the channel.

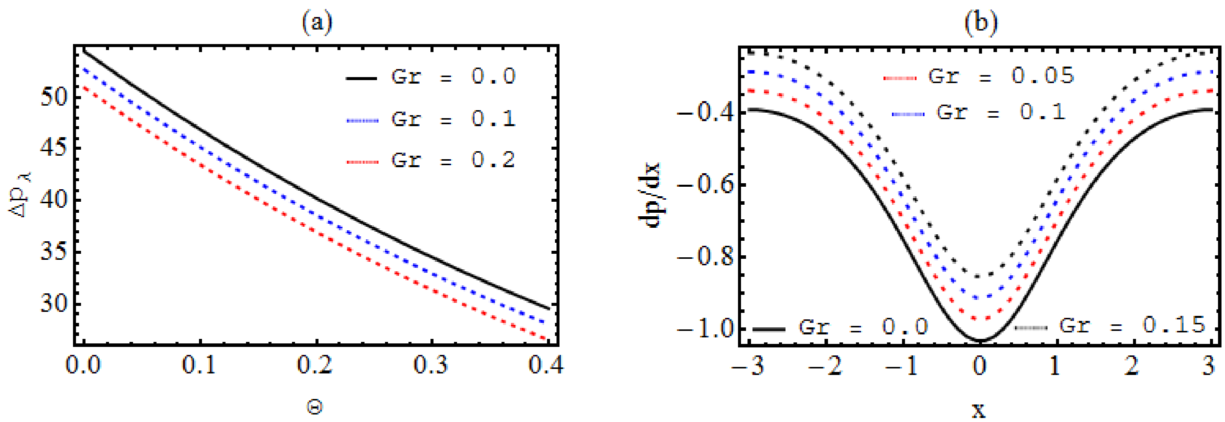


Fig.4 Effects of Gr on pressure rise per wavelength Δp_λ versus flow rate Θ (a) and longitudinal pressure gradient dp/dx for $\Theta = 0.8$ When $\varphi = 0.3,\ \alpha_1 = 0.5,\ \alpha_2 = 2.0,\ Br = 1.0$

Fig. 4 elucidated the effects of thermal Grashof number on the pressure rise per wavelength Δp_λ and on pressure gradient dp/dx. As we increase the *Gr* the pressure difference shows a notable decrease. This means that huge buoyancy effects results in the decrease of the pumping efficiency. Axial pressure gradient shows an enormous increase for the increasing values of the Gr and these effects are dominant at the inlet and outlet of the channel. Furthermore axial pressure gradient dp/dx reaches its maximum value in magnitude at the center of the channel.

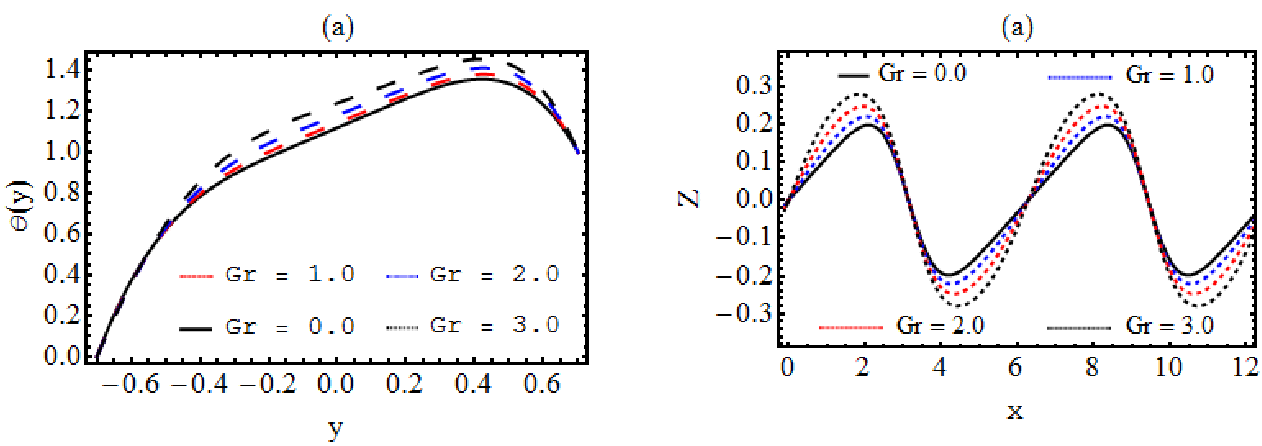


Fig.5 Effects of Gr on temperature profile θ (a) heat transfer coefficient Z (b) When $\varphi = 0.3,\ \alpha_1 = 0.5,\ \alpha_2 = 2.0,\ Br = 1.0, x = \pi$

Effects of Grashof number on the internal temperature and the heat flux (heat transfer coefficient) at the walls of the channel are given in Fig. 5. Due to the increase in the Gr the effects of the gravity forces increases than the viscous forces. This results in the increase in the temperature of the fluid and the amplitude of the heat transfer coefficient Z at the upper wall of the channel also increases.

4. Comparison

The boundary value problem is solved numerically using implicit finite difference method along iterative method with a code is executed in Fortran and graphical and tabulated results for both the solutions are presented in this subsection.

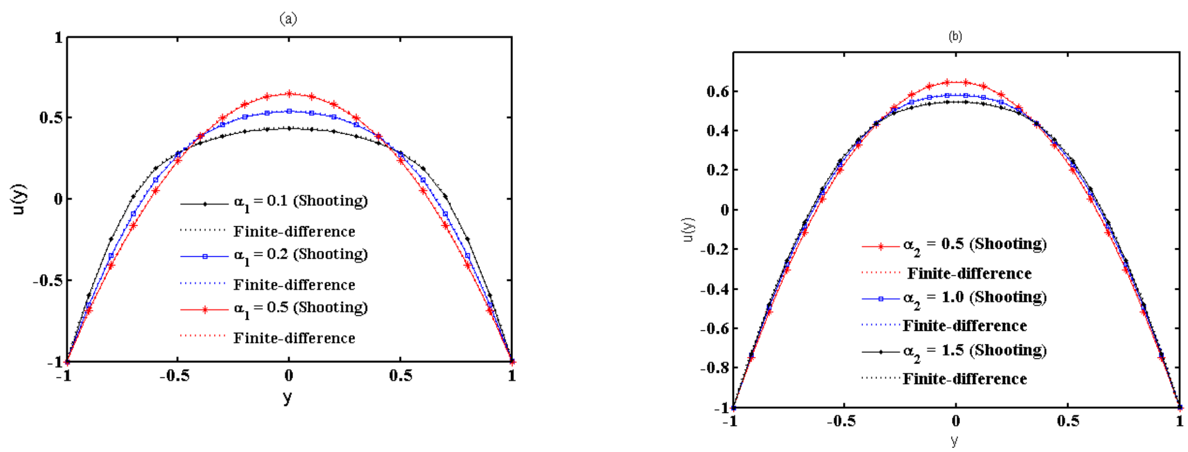
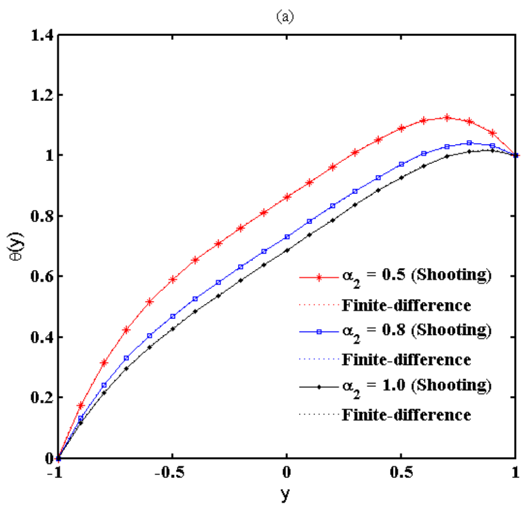


Fig.6 Comparison of velocity profile for different values of α_1 when $\alpha_2 = 0.5$ (a) for different values of α_2 when $\alpha_1 = 0.5$ (b) other parameters are $Br = 0.5, Gr = 0.0, x = \pi/2$

In order to make a comparison of the velocity profile Fig. 6 is plotted and it is observed that for both shooting and finite difference methods results are strongly agree with each other and these results shows a good agreement with Ali et al.[21]. Also when $\alpha_1 = \alpha_2$ the results for viscous fluid are obtained.



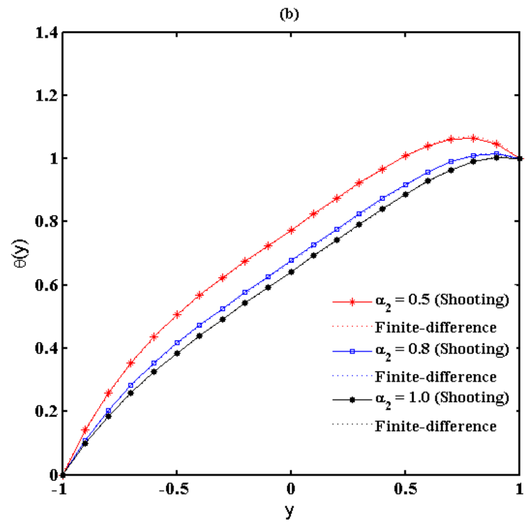


Fig.7 Comparison of temperature profile for different values of α_2 when $Gr = 0$ (a) for different values of α_2 when $Gr = 1.0$ (b) where $\alpha_1 = 0.5$ and $x = \pi/2$

In Fig.7(a) temperature profile is plotted when the buoyancy forces are zero and results for both the techniques strongly coincide with each other for the increasing values of non-Newtonian fluid parameter α_2. Also under the influence of buoyancy forces the temperature profiles show good agreement.

Table 01: Comparison of axial velocity $u(y)$ for different parameters

Gr	Br	α_1	α_2	$\eta = 0$		$\eta = h/4$		$\eta = h/2$	
				Shooting	FDM	Shooting	FDM	Shooting	FDM
0	1.0	0.5	0.5	0.650000	0.6505	0.546875	0.5460	0.23750	0.2369
		0.4		0.627151	0.6272	0.537655	0.5378	0.245563	0.2452
		0.3		0.593851	0.5932	0.520873	0.5206	0.257903	0.2578
0.1		0.5		0.650868	0.6501	0.549455	0.5490	0.24057	0.2400
	1.5			0.651304	0.6507	0.545856	0.5481	0.234274	0.2362
	1.0		0.8	0.605767	0.6053	0.532817	0.5324	0.258574	0.2584
	0.3		1.0	0.584264	0.5839	0.523821	0.5242	0.268003	0.2681
	0.2		0.5	0.650173	0.6808	0.240614	0.2442	0.548953	0.5501
0.3	1.0		1	0.586625	0.5860	0.531005	0.5308	0.279212	0.2790

Table 02: Comparison of temperature profile $\theta(y)$ for different parameters

Gr	Br	α_1	α_2	$\eta = 0$		$\eta = h/4$		$\eta = h/2$	
				Shooting	FDM	Shooting	FDM	Shooting	FDM
0	1.0	0.5	0.5	1.4075	1.4074	1.52896	1.5286	1.60078	1.6007
		0.4		1.24133	1.2412	1.36378	1.3636	1.44809	1.4479
		0.3		1.07324	1.00732	1.19659	1.1963	1.29337	1.2932
0.1		0.5		1.40967	1.4092	1.53101	1.5291	1.60311	1.6028

	1.5		1.86613	1.8611	1.73592	1.7350	1.52981	1.5291
	1.0	0.8	1.08488	1.0863	1.20799	1.2078	1.30359	1.3033
	0.3	1.0	0.641002	0.6409	0.884097	0.8835	0.76588	0.7650
	0.2	0.5	0.68159	0.6808	0.92032	0.9233	0.805861	0.8052
0.3	1.0		1.5890	1.5885	1.70975	1.7094	1.77094	1.7675

From Table 01 and Table 02 both the velocity and temperature profiles for different involved parameters is similar for both the methods.

5. Concluding remarks

Mixed convective peristaltic transport of an Oldroyd 4-constant fluid in a planner channel were investigated in this article. The main finding of the present communication are

- The velocity profile decreases in the left half of the channel whereas in the right half of the channel the effects are quite opposite with increase in thermal Grashof number.
- In the presence of gravity and viscous forces the stream function shows decreasing behaviors.
- By increasing thermal Grashof number the size of trapping bolus increased.
- The pressure rise per wavelength decreases while the pressure gradient increases by increasing thermal Grashof number.
- With out buoyancy forces the velocity profile show a good agreement with the available results.

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