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Research Article

Impacts of time varying wall temperature and concentration on MHD free convective flow of a rotating fluid due to moving free-stream with hall and ion-slip currents

Jitendra Kumar Singh[1], Gauri Shenkar Seth[2], Pratima Rohidas[1]

[1]*Department of Mathematics, Vijayanagara Sri Krishnadevaraya University, Ballari-583105, India*
[2]*Department of Applied Mathematics, Indian Institute of Technology (ISM), Dhanabad-826004, India*

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Abstract

Present paper mathematically discusses unsteady MHD free convection flow of thermally conducting, chemically reacting, and rotating fluid over a vertical plate due to moving free-stream. The applied magnetic field is considered to be strong enough to generate Hall and ion-slip currents. The wall temperature and concentration are considered to be linearly varying with time. Laplace transform technique is successfully implemented to solve the resulting partial differential equations representing the fluid motion. The expression for fluid velocity is derived in four special cases., i.e. (i) for those fluids whose viscosity, thermal diffusivity and molecular diffusivity are not of same order of magnitude, (ii) for those fluids whose viscosity and thermal diffusivity are of same order of magnitude while molecular diffusivity is of different order, (iii) for those fluids whose viscosity and molecular diffusivity are of same order of magnitude while thermal diffusivity is of different order, and (iv) for those fluids whose viscosity, thermal diffusivity and molecular diffusivity are of same order of magnitude. To discuss the specific features of the flow, numerical computation is carried out. The variations in fluid velocity, fluid temperature and concentration are presented through graphs whereas skin friction coefficient, rate of heat and mass transfer are presented in tables. A notable observation recorded that in the absence of thermal buoyancy force there appears reverse flow in the secondary flow direction while in the absence of concentration buoyancy force there does not exists reverse flow in the secondary flow direction.

Keywords: Free convection; Free-stream; Hall and ion-slip currents; Rotation.

Corresponding email: s.jitendrak@yahoo.com (Jitendra Kumar Singh)

1. Introduction

Study of free or natural convection flow is significant because it is frequently encountered by the engineers and scientists in many natural, technological and biological systems. It is widely accepted that it arises due to variation is density in the field of gravity. The variation in density may occur either due to non-uniform heating or due non-uniform species distributions or due to both. Free convection is a process of heat and mass transfer in fluid. Convective heat transfer may find significant engineering and industrial applications such as heat exchangers, thermal insulators, electronic equipment cooling systems etc. Convective mass transfer arises in many biological and chemical processes such as absorption, drying, precipitation, distillation, filtration etc. Stimulated form enormous industrial applications many researchers studied free convective flow problems considering different geometries by using various analytical and numerical methods. Some relevant contributions to the free convective flows are due Hsieh et al. (1993), Vasseur and Degan (1998), Bachok et al. (2010), Chang et al. (2011), Khan et al. (2014), Hayat et al. (2014), Kamran et al. (2014), and Das et al. (2015). Motion of electrically conducting fluid in the presence of an applied magnetic field produces a flow controlling force. This flow controlling force plays a prominent role in determination of flow behavior. This flow controlling nature is significant in boundary layer control. In biological system, it is important in reduction of blood flow rate in arterial system, which is useful in treatment of some cardiovascular disorders and in the problems which increase the rate of circulation of blood such as haemorrhage and hypertension. MHD free convective flow of electrically and thermally conducting and chemically reacting fluid is an enormously growing topic of research due to its presence in natural, technological and biological systems and industrial applications viz. nuclear engineering, chemical industries, aerodynamics and polymer processing. Rahman and Salahuddin (2010) studied the heat and mass transfer behaviour of hydromagnetic flow over an inclined surface with variable viscosity and electrical conductivity. Rajput and Kumar (2012) discussed the influence of heat radiation on unsteady MHD boundary layer flow over an impulsively moving vertical plate with time varying wall temperature and concentration. Moreover, Narhari and Debnath (2013) presented the MHD free convective flow over an accelerated vertical plate with heat source/sink. Butt and Ali (2016) analysed the entropy generation on hydromagnetic free convective flow over an oscillating plate. Das et al. (2016) studied the Hall current influence on unsteady MHD free convective flow over an impulsively moving surface by considering ramped wall temperature and concentration. They found that ramped wall temperature significantly affecting the velocity profiles. Permeability of the porous medium exerts a flow controlling force on the fluid flows which is like the flow controlling force induced by magnetic field. In recent years, study of MHD free convective flow through the porous medium has attracted the attention from several researchers due to its various industrial and technological applications such as extraction of petroleum products from exploration wells, geothermal energy extraction, metallurgy of metals etc. Balamurugan et al. (2015) analysed the influence of time dependent suction and chemical reaction on unsteady MHD free convective flow over a moving vertical plate embedded in porous medium. Subsequently, Rama Mohan Reddy et al. (2016) investigated fully developed unsteady MHD free convective flow of rotating fluid through porous medium over an infinite vertical porous plate. They found that Schemidt number suppresses the momentum boundary layer thickness. Singh et al. (2016) analysed the influence of Hall current and rotation on unsteady MHD free convective flow over an exponentially accelerated vertical with ramped wall temperature and fluctuating concentration. They noticed in their investigation that thermal diffusion tends to raise the momentum and thermal boundary layer thicknesses. Thermal radiation effects on MHD free convection flow over a vertical plate implanted in porous medium with ramped wall temperature is presented by Pandit et al. (2017). It is seen that the radiation parameter tends to reduce the thermal boundary layer thickness. Recently, Ali et al. (2017) discussed unsteady MHD boundary layer flow of a second grade fluid over an oscillating plate embedded in porous medium with thermal radiation effect. Some significant contributions to the topic are also due to El-Kabeir et al. (2007), Beg et al. (2009), Das (2011), Seth et al. (2011), Seth and Sarkar (2015), Hossain et al. (2015) and Hussain et al. (2017). It is praiseworthy to note that in some industrial and technological applications applied magnetic field is strong enough to produce Hall and ion-slip currents. In such situations combined influence of Hall and ion-slip currents cannot be neglected from MHD fluid flows. The combined effects of Hall and ion-slip currents on unsteady MHD boundary layer flow past a semi-infinite vertical plate are discussed by Abo-Eldahab and Aziz (2000), Hossain et al. (2015), Singh et al. (2017) and Singh and Srinivasa (2018). In the above mentioned research investigations unsteadiness in the boundary layer flow appears due to movement of the plate. Lighthill (1954) was firstly investigated the response of fluctuation of free-stream to the two-dimensional boundary layer flow about a cylindrical body because fluctuation of free-stream arises in some technological systems. He observed that maximum of the skin friction at any point anticipate the maximum of the free-stream velocity. Later on, Messiha (1965) implemented this idea in investigation of two-dimensional oscillatory boundary layer flow over an infinite flat porous plate. Patil and Roy (2010) discussed the influence of heat generation/absorption on unsteady mixed convective boundary layer flow due to moving vertical plate and free-stream. Sarkar and Seth (2017) analysed the influence of Hall current, rotation and heat absorption on unsteady MHD free convective flow over a vertical plate embedded in porous medium due to impulsive and accelerated movement of the free-stream whereas Seth et al. (2017) discussed this problem for exponentially accelerated free stream. An interesting result noted in their studies that reverse flow arises in the secondary flow direction due to presence of thermal buoyancy force. Recently, Singh et al. (2017) presented an analytical study of unsteady MHD free convective flow of a rotating fluid induces due of fluctuation of the free-stream considering Hall and ion-slip currents into account. They found that reverse flow appears in the secondary flow direction even in the absence of thermal and solutal buoyancy forces. Similar observation recorded in the investigation of Singh et al. (2018). However the impacts of time vary wall temperature and concentration are not considered in these investigations which may appear in some technological systems. The motive in this paper is to study the impacts of time varying wall temperature and concentration on unsteady MHD free convective flow of thermally conducting, chemically reacting and rotating fluid over a vertical plate due to exponentially accelerated free-stream with rotation, Hall and ion-slip currents. The mathematical model of the present problem is governed by the coupled partial differential equations and it is solved by using Laplace transform method. A remarkable observation recorded from the present problem that in the absence of thermal buoyancy force there appears reverse flow in the secondary flow direction while in the absence of concentration buoyancy force there does not exists reverse flow in the secondary flow direction.

2. Mathematical Modeling of the Problem

Let us consider the free convection boundary layer flow of a thermally conducting, chemically reacting and rotating fluid over a vertical plate under the action of a strong applied magnetic field. Reference system for the flow is chosen in such a way that $x'z'$ -plane is coincident with the plate and y' -axis is normal to it. The flow system is considered to be rotating uniformly with angular velocity $\vec{\Omega}$ about the axis normal to the plate and magnetic field B_0 is applied along the axis of rotation. At initial stage $t' \leq 0$, the entire flow system is stationary and plate temperature and concentration are considered to be T'_∞ and C'_∞ respectively. Suddenly, when $t' > 0$, the plate temperature and concentration varying linearly with the time while plate remains stationary. The free-stream is set up into motion with velocity $U_0 e^{U_0^2 t'/\upsilon}$, where U_0 is uniform velocity while free-stream temperature and concentration are remains unchanged. In the present analysis all flow variables depend on y' and t' only. In this analysis fluid is considered to be such that its magnetic diffusivity is very large, so induced magnetic field can be neglected (Sutton and Sherman (1965)). Further, it is assumed that the boundary layer and Boussinesq approximations holds good.

In the essence of above made assumptions, and approximations the system of equations describing free convective boundary layer flow of a rotating fluid over a vertical plate with Hall and ion-slip currents are given by (Seth et al. (2016) and Singh et al. (2017))

the momentum equations:

$$\frac{\partial u'}{\partial t'} + 2\Omega w' = \frac{\partial U}{\partial t'} + \upsilon \frac{\partial^2 u'}{\partial y'^2} + \frac{\sigma B_0{}^2}{\rho}\left[\frac{(U-u')\alpha_e - w'\beta_e}{\alpha_e^2+\beta_e^2}\right] - \frac{\upsilon'(U-u')}{k'} + g\beta'_T(T'-T'_\infty) + g\beta'_C(C'-C'_\infty), \tag{1}$$

$$\frac{\partial w'}{\partial t'} - 2\Omega(u'-U) = \upsilon \frac{\partial^2 w'}{\partial y'^2} - \frac{\sigma B_0{}^2}{\rho}\left[\frac{w'\alpha_e + (U-u')\beta_e}{\alpha_e^2+\beta_e^2}\right] - \frac{\upsilon' w'}{k'}, \tag{2}$$

the energy equation with heat source:

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{Q_0}{\rho C_p}(T'-T'_\infty) \tag{3}$$

the concentration equation with chemical reaction:

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - K'(C'-C'_\infty) \tag{4}$$

where $\alpha_e = 1 + \beta_i \beta_e$ and $\upsilon, \rho, \sigma, \beta_i, \beta_e, g, \beta_T', \beta_C', T', C', k', k, C_p, Q_0, D$ and K' are respectively, kinematic coefficient of viscosity, fluid density, electrical conductivity, ion-slip parameter, Hall current parameter, acceleration due to gravity, volumetric coefficient of thermal expansion, volumetric coefficient of species concentration expansion, fluid temperature, species concentration, permeability of the porous medium, thermal conductivity of the fluid, specific heat at constant pressure, heat source coefficient, molecular diffusivity and chemical reaction parameter.

The Initial and boundary conditions to be satisfied are given by

$$t' \le 0: \; u' = w' = 0, \; T' = T'_\infty, \; C' = C'_\infty, \qquad \text{for } y' \ge 0, \tag{5}$$

$$t' > 0: \begin{cases} u' = w' = 0, \;\; T' = T'_\infty + \left(T'_w - T'_\infty\right)\dfrac{U_0^2 t'}{\upsilon}, \; C' = C'_\infty + \left(C'_w - C'_\infty\right)\dfrac{U_0^2 t'}{\upsilon}, & \text{at } y' = 0, \\ u' \to U\left(t'\right) = U_0 e^{U_0^2 t'/\upsilon}, \; w' \to 0, T' \to T'_\infty, C' \to C'_\infty, & \text{as } y' \to \infty. \end{cases} \tag{6}$$

Following non-dimensional variables are introduced to transform the dimensional governing equations (1)-(4) and initial and boundary conditions (5) and (6) into non-dimensional form

$$y = \frac{y' U_0}{\upsilon}, \; u = \frac{u'}{U_0}, w = \frac{w'}{U_0}, t = \frac{t' U_0^2}{\upsilon}, T = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \; C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \; f = \frac{U}{U_0}, \; q = u + iw.$$

On using the above defined transformation the flow governing equations (1)-(4) reduces to the following non dimensional equations

$$\frac{\partial q}{\partial t} = \frac{\partial f}{\partial t} + \frac{\partial^2 q}{\partial y^2} + X_3\left(f - q\right) + G_T T + G_C C \tag{7}$$

$$\frac{\partial T}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2} - \phi T \tag{8}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - K_1 C \tag{9}$$

where $X_1 = (1/k_1) + \left(M\alpha_e / \alpha_e^2 + \beta_e^2\right)$, $X_2 = (2E) + \left(M\beta_e / \alpha_e^2 + \beta_e^2\right)$, $X_3 = X_1 - iX_2$, $M = \sigma B_0^2 \upsilon / \rho U_0^2$ is magnetic parameter, $E = \Omega\upsilon / U_0^2$ is rotation parameter, $k_1 = k' U_0^2 / \upsilon^2$ is permeability parameter, $G_T = g\beta_T' \upsilon\left(T'_w - T'_\infty\right)/U_0^3$ is thermal Grashof number, $G_C = g\beta_C'\upsilon\left(C'_w - C'_\infty\right)/U_0^3$ is solutal Grashof number, $\Pr = \rho\upsilon C_p / k$ is Prandtl number, $\phi = Q_0\upsilon / \rho C_p U_0^2$ is heat absorption parameter, $Sc = \upsilon / D$ is Schmidt number, and $K_1 = K'\upsilon / U_0^2$ is chemical reaction parameter.

The non-dimensional initial and boundary conditions are given by

$$t \le 0 : q = T = C = 0, \qquad \text{for } y \ge 0, \tag{10}$$

$$t > 0 : \begin{cases} q = 0,\, T = C = t, & \text{at } y = 0, \\ q \to f(t) = e^t,\ T, C \to 0, & \text{as } y \to \infty. \end{cases} \tag{11}$$

Simultaneous system of partial differential equations (7)-(9) represents the mathematical model of the present problem subject to the initial and boundary conditions (10) and (11).

3. Solution of the Problem

To solve equations (7)-(9) subject to initial and boundary conditions (10) and (11) the Laplace transform method is used. The solution for fluid velocity is obtained in the following four cases:

Case (i): When viscosity thermal diffusivity and molecular diffusivity of the fluid are not of same order of magnitude i.e., $\Pr \neq 1$ and $Sc \neq 1$, the fluid velocity is given by

$$\begin{aligned} &q(y,t) \\ &= e^t - F_1(y,t,1,X_3,1) - \frac{G_T}{(\Pr-1)X_4^2}\Big[\big\{F_1(y,t,1,X_3,0) - X_4 F_2(y,t,1,X_3,0) - F_1(y,t,1,X_3,-X_4)\big\} \\ &-\big\{F_1(y,t,\Pr,\phi,0) - X_4 F_2(y,t,\Pr,\phi,0) - F_1(y,t,\Pr,\phi,-X_4)\big\}\Big] \\ &-\frac{G_C}{(Sc-1)X_5^2}\Big[\big\{F_1(y,t,1,X_3,0) - X_5 F_2(y,t,1,X_3,0) - F_1(y,t,1,X_3,-X_5)\big\} \\ &-\big\{F_1(y,t,Sc,K_1,0) - X_5 F_2(y,t,Sc,K_1,0) - F_1(y,t,Sc,K_1,-X_5)\big\}\Big], \end{aligned} \tag{12}$$

where $X_4 = (\Pr\phi - X_3)/(\Pr-1)$, $X_5 = (ScK_1 - X_3)/(Sc-1)$,

$$F_1(y,t,a,b,c) = \frac{e^{ct}}{2}\left[e^{y\sqrt{a(b+c)}} erfc\left(\frac{y}{2}\sqrt{\frac{a}{t}} + \sqrt{(b+c)t}\right) + e^{-y\sqrt{a(b+c)}} erfc\left(\frac{y}{2}\sqrt{\frac{a}{t}} - \sqrt{(b+c)t}\right)\right],$$

$$\begin{aligned} F_2(y,t,a,b,c) = \frac{e^{ct}}{2}\Bigg[&\left(t + \frac{y}{2}\sqrt{\frac{a}{b+c}}\right)e^{y\sqrt{a(b+c)}} erfc\left(\frac{y}{2}\sqrt{\frac{a}{t}} + \sqrt{(b+c)t}\right) \\ &+\left(t - \frac{y}{2}\sqrt{\frac{a}{b+c}}\right)e^{-y\sqrt{a(b+c)}} erfc\left(\frac{y}{2}\sqrt{\frac{a}{t}} - \sqrt{(b+c)t}\right)\Bigg]. \end{aligned}$$

Case (ii): When viscosity and thermal diffusivity of the fluid are of same order of magnitude while molecular diffusivity is of different order i.e., $\Pr = 1, Sc \neq 1,$ the fluid velocity is expressed as

$$q(y,t) = e^t - F_1\left(y,t,1,X_3,1\right) + \frac{G_T}{\left(\phi - X_3\right)}\left[F_2\left(y,t,1,X_3,0\right) - F_2\left(y,t,1,\phi,0\right)\right]$$
$$-\frac{G_C}{\left(Sc-1\right)X_5^2}\left[\left\{F_1\left(y,t,1,X_3,0\right) - X_5 F_2\left(y,t,1,X_3,0\right) - F_1\left(y,t,1,X_3,-X_5\right)\right\}\right.$$
$$\left.-\left\{F_1\left(y,t,Sc,K_1,0\right) - X_5 F_2\left(y,t,Sc,K_1,0\right) - F_1\left(y,t,Sc,K_1,-X_5\right)\right\}\right] \tag{13}$$

Case (iii): When viscosity and molecular diffusivity of the fluid are same order of magnitude while thermal diffusivity is of different order i.e., $\Pr \neq 1,\ Sc = 1,$ the expression for fluid velocity is,

$$q\left(y,t\right) = e^t - F_1\left(y,t,1,X_3,1\right) - \frac{G_T}{\left(\Pr-1\right)X_4^2}\left[\left\{F_1\left(y,t,1,X_3,0\right) - X_4 F_2\left(y,t,1,X_3,0\right) - F_1\left(y,t,1,X_3,-X_4\right)\right\}\right.$$
$$\left.-\left\{F_1\left(y,t,\Pr,\phi,0\right) - X_4 F_2\left(y,t,\Pr,\phi,0\right) - F_1\left(y,t,\Pr,\phi,-X_4\right)\right\}\right]$$
$$+\frac{G_C}{\left(K_1 - X_3\right)}\left[F_2\left(y,t,1,X_3,0\right) - F_2\left(y,t,1,K_1,0\right)\right]. \tag{14}$$

Case (iv): When viscosity, thermal diffusivity and molecular diffusivity are same order of magnitude i.e., $\Pr = 1,\ Sc = 1,$ the fluid velocity is

$$q(y,t) = e^t - F_1\left(y,t,1,X_3,1\right) + \frac{G_T}{\left(\phi - X_3\right)}\left[F_2\left(y,t,1,X_3,0\right) - F_2\left(y,t,1,\phi,0\right)\right]$$
$$+\frac{G_C}{\left(K_1 - X_3\right)}\left[F_2\left(y,t,1,X_3,0\right) - F_2\left(y,t,1,K_1,0\right)\right]. \tag{15}$$

The solution for fluid temperature and species concentration are, respectively, expressed as follows

$$T\left(y,t\right) = F_2\left(y,t,\Pr,\phi,0\right) \tag{16}$$

$$C\left(y,t\right) = F_2\left(y,t,Sc,K_1,0\right) \tag{17}$$

4. Skin Friction Coefficient and Rate of Heat and Mass Transfer at the Plate

The skin friction coefficient for the fluid whose viscosity, thermal diffusivity and molecular diffusivity are not of same order of magnitude, is expressed as

$$\begin{aligned}\tau &= \left(\tau_x + i\tau_z\right)\\ &= F_3\left(t,1,X_3,1\right) + \frac{G_T}{\left(\mathrm{Pr}-1\right)X_4^2}\Big[\left\{F_3\left(t,1,X_3,0\right) - X_4F_4\left(t,1,X_3,0\right) - F_3\left(t,1,X_3,-X_4\right)\right\}\\ &\quad -\left\{F_3\left(t,\mathrm{Pr},\phi,0\right) - X_4F_4\left(t,\mathrm{Pr},\phi,0\right) - F_3\left(t,\mathrm{Pr},\phi,-X_4\right)\right\}\Big]\\ &\quad + \frac{G_C}{\left(Sc-1\right)X_5^2}\Big[\left\{F_3\left(t,1,X_3,0\right) - X_5F_4\left(t,1,X_3,0\right) - F_3\left(t,1,X_3,-X_5\right)\right\}\\ &\quad -\left\{F_3\left(t,Sc,K_1,0\right) - X_5F_4\left(t,Sc,K_1,0\right) - F_3\left(t,Sc,K_1,-X_5\right)\right\}\Big]\end{aligned} \tag{18}$$

where

$$F_3\left(t,a,b,c\right) = e^{ct}\left[\sqrt{a\left(b+c\right)}\left\{erfc\sqrt{\left(b+c\right)t} - 1\right\} - \sqrt{\frac{a}{\pi t}}e^{-(b+c)t}\right],$$

$$F_4\left(t,a,b,c\right) = e^{ct}\left[t\left(\sqrt{a\left(b+c\right)} + 1/2\sqrt{\frac{a}{b+c}}\right)\left\{erfc\sqrt{\left(b+c\right)t} - 1\right\} - \sqrt{\frac{at}{\pi}}e^{-(b+c)t}\right].$$

The skin friction coefficient for the fluid whose viscosity and thermal diffusivity are same order of magnitude while molecular diffusivity is of different order, is given by

$$\begin{aligned}\tau &= \left(\tau_x + i\tau_z\right)\\ &= F_3\left(t,1,X_3,1\right) - \frac{G_T}{\left(\phi - X_3\right)}\left[F_4\left(t,1,X_3,0\right) - F_4\left(t,1,\phi,0\right)\right]\\ &\quad + \frac{G_C}{\left(Sc-1\right)X_5^2}\Big[\left\{F_3\left(t,1,X_3,0\right) - X_5F_4\left(t,1,X_3,0\right) - F_3\left(t,1,X_3,-X_5\right)\right\}\\ &\quad -\left\{F_3\left(t,Sc,K_1,0\right) - X_5F_4\left(t,Sc,K_1,0\right) - F_3\left(t,Sc,K_1,-X_5\right)\right\}\Big]\end{aligned} \tag{19}$$

The expression for the skin friction coefficient for the fluid whose viscosity and thermal diffusivity are of same order of magnitude while molecular diffusivity is of different order is

$$\begin{aligned}\tau &= \left(\tau_x + i\tau_z\right)\\ &= F_3\left(t,1,X_3,1\right) + \frac{G_T}{\left(\mathrm{Pr}-1\right)X_4^2}\Big[\left\{F_3\left(t,1,X_3,0\right) - X_4F_4\left(t,1,X_3,0\right) - F_3\left(t,1,X_3,-X_4\right)\right\}\\ &\quad -\left\{F_3\left(t,\mathrm{Pr},\phi,0\right) - X_4F_4\left(t,\mathrm{Pr},\phi,0\right) - F_3\left(t,\mathrm{Pr},\phi,-X_4\right)\right\}\Big]\\ &\quad - \frac{G_C}{\left(K_1 - X_3\right)}\left[F_4\left(t,1,X_3,0\right) - F_4\left(t,1,K_1,0\right)\right]\end{aligned} \tag{20}$$

The skin friction coefficient for the fluid whose viscosity, thermal diffusivity and molecular diffusivity are of same order of magnitude is

$$\tau = \left(\tau_x + i\tau_z\right)$$
$$= F_3\left(t,1,X_3,1\right) - \frac{G_T}{\left(\phi - X_3\right)}\left[F_4\left(t,1,X_3,0\right) - F_4\left(t,1,\phi,0\right)\right] \tag{21}$$
$$-\frac{G_C}{\left(K_1 - X_3\right)}\left[F_4\left(t,1,X_3,0\right) - F_4\left(t,1,K_1,0\right)\right]$$

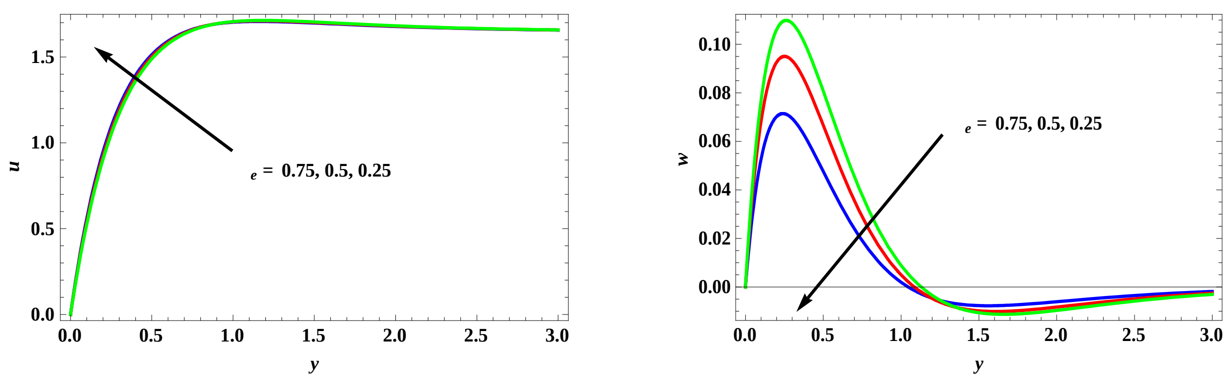
The Nusselt and Sherwood numbers, which express the rate of heat and mass transfers at the plate respectively, are presented as

$$Nu = -F_4\left(t,\Pr,\phi,0\right) \tag{22}$$

$$Sh = -F_4\left(t,Sc,K_1,0\right) \tag{23}$$

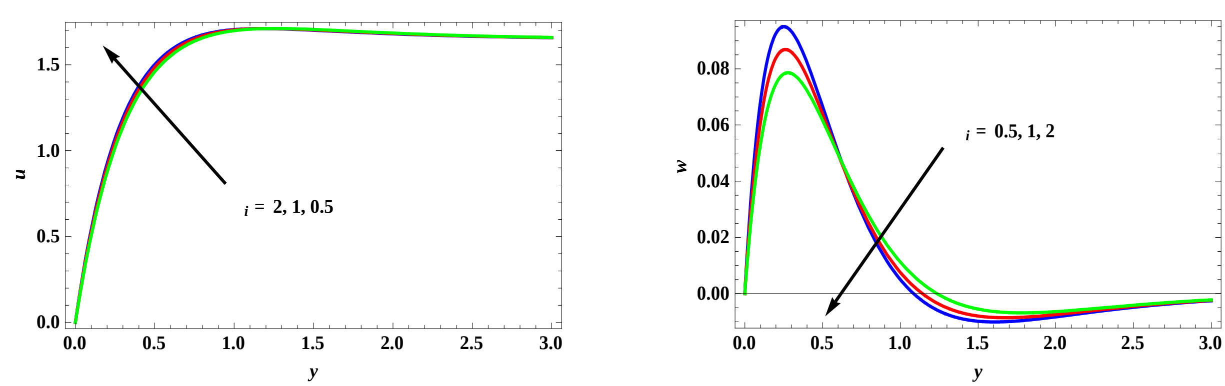
5. Results and Discussion

A numerical computation has been performed to get the impacts of various system parameters on velocity profiles, temperature profiles, concentration profiles, skin friction coefficient and rate of heat and mass transfer at the plate. The velocity profiles for the fluid are presented in Figs. 1-12 whereas Figs. 13-16 and Figs. 16-18, respectively, demonstrate the temperature and concentration profiles. The change in flow behavior corresponds to change in Hall and ion-slip currents are displayed in the Figs. 1 and 2. It can be easily noticed that both Hall and ion-slip currents have tendency to suppress the primary flow in the neighboring boundary layer region of the plate while this tendency is upturned near the free-stream. This may be due to the fact that flow induces due to moving free-stream along the primary flow direction. Hall current tends to raise the secondary flow in the boundary layer regions adjacent to plate and free-steam, which is the usual tendency of Hall current to induce secondary flow. Ion-slip current shows the opposite behavior as that of Hall current on the secondary flow. The action of Coriolis force and magnetic force on the fluid flow are demonstrated in Figs. 3 and 4. Both the Coriolis and magnetic forces tend to raise primary flow in the boundary layer region adjacent to the plate while this tendency is upturned in the boundary layer region near the free-stream. Both Coriolis and magnetic forces have tendency to raise secondary flow in the boundary layer region adjacent to the plate and free-stream. In general Coriolis force has tendency to induce secondary flow, our result is in agreement with this. Darcian drag force effects are presented in Fig. 5. Daracian drag force is inversely proportional to the permeability of the porous medium. Daracian drag force shows similar behavior as that of magnetic force on the primary flow and it shows the reverse behavior as that of magnetic force on the secondary flow. The Daracian drag force has flow controlling nature. Our result also comply it except in the boundary layer region adjacent to the plate, which may be due to the fact that the flow is induced due to moving free-stream. Buoyancy forces effects on flow behavior are illustrated in Figs. 6 and 7. Primary flow gets raised on rising thermal and solutal Grashof numbers. Since rise in thermal and solutal Grashof numbers give rise in thermal and concentration buoyancy effects respectively. This concludes that thermal and concentration buoyancy forces tend to raise primary flow. It is also seen that reverse flow induces in the secondary flow direction. On rising the buoyancy forces, secondary flow fall down in the boundary layer region adjacent to the plate while this tendency is upturned in the boundary layer region adjacent to the free-stream. A remarkable observation recorded that in the absence of thermal buoyancy force there appears reverse flow in the secondary flow direction while in the absence of concentration buoyancy force there does not exist reverse flow in the secondary flow direction. Influence of progression of time on the flow behavior is displayed in Fig. 8. It can be easily noticed that primary flow gets raised as time progresses. As time progresses secondary flow get enhanced in the boundary layer region in the vicinity of the plate and free-stream. Figs. 9 and 11, respectively, represent the consequences of thermal and mass diffusions on the fluid flow. Primary flow get reduced on raising Prandtl and Schmidt numbers, which indicates that thermal and mass diffusions tend to rise the primary flow. Reverse flow induces in the secondary flow direction and these have similar influences as that of buoyancy forces on the secondary flow. Variation in the flow behavior corresponds to heat absorption is shown in the Fig. 10 whereas Fig. 12 represents variation in flow behavior corresponds to chemical reaction. It is observed that primary flow fall down on raising the heat absorption and chemical reaction respectively. Reverse flow exists in the secondary flow direction and heat absorption and chemical reaction show the opposite nature as that of buoyancy forces on the secondary flow.



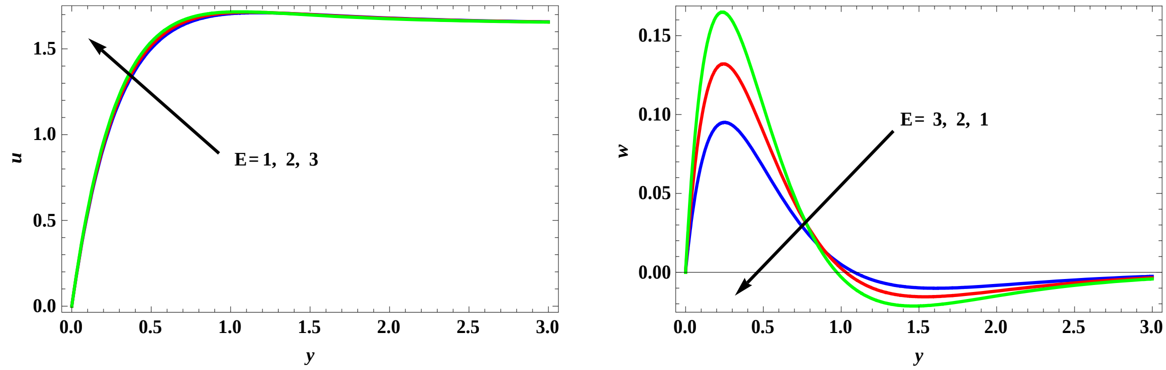
(a) (b)

Fig.1 Velocity profiles in the (a) primary and (b) secondary flow directions when $\beta_i = 0.5, E = 1, M = 9, k_1 = 0.3, G_T = 4, G_C = 5, t = 0.5, \Pr = 0.71, \phi = 1, Sc = 0.22, K_1 = 0.2.$



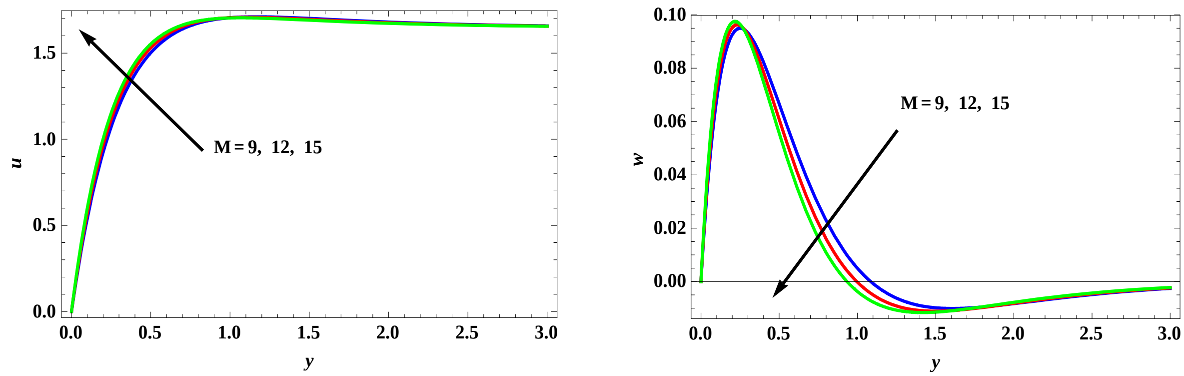
(a) (b)

Fig.2 Velocity profiles in the (a) primary and (b) secondary flow directions when $\beta_e = 0.5,\ E = 1, M = 9,\ k_1 = 0.3,\ G_T = 4,\ G_C = 5,\ t = 0.5, \Pr = 0.71,\ \phi = 1, Sc = 0.22, K_1 = 0.2.$



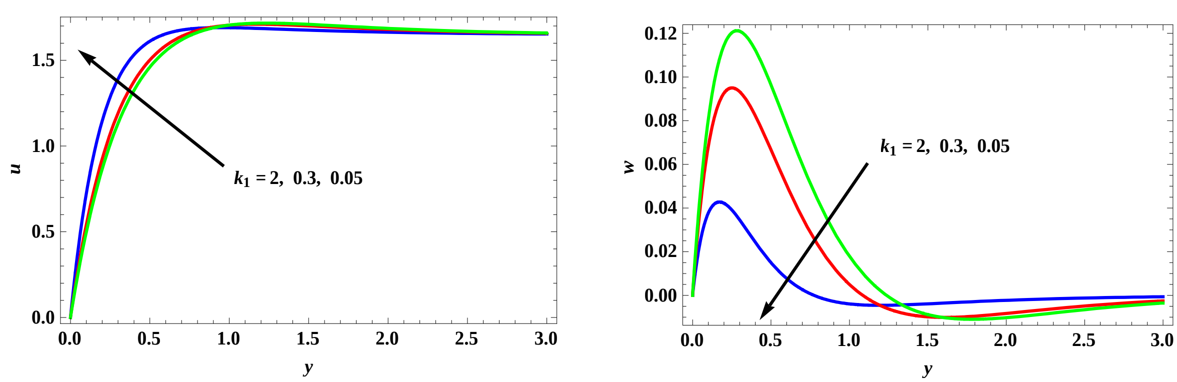
(a) (b)

Fig.3 Velocity profiles in the (a) primary and (b) secondary flow directions when $\beta_e = 0.5, \beta_i = 0.5, M = 9, k_1 = 0.3, G_T = 4, G_C = 5,\ t = 0.5, \Pr = 0.71,\ \phi = 1, Sc = 0.22,\ K_1 = 0.2.$



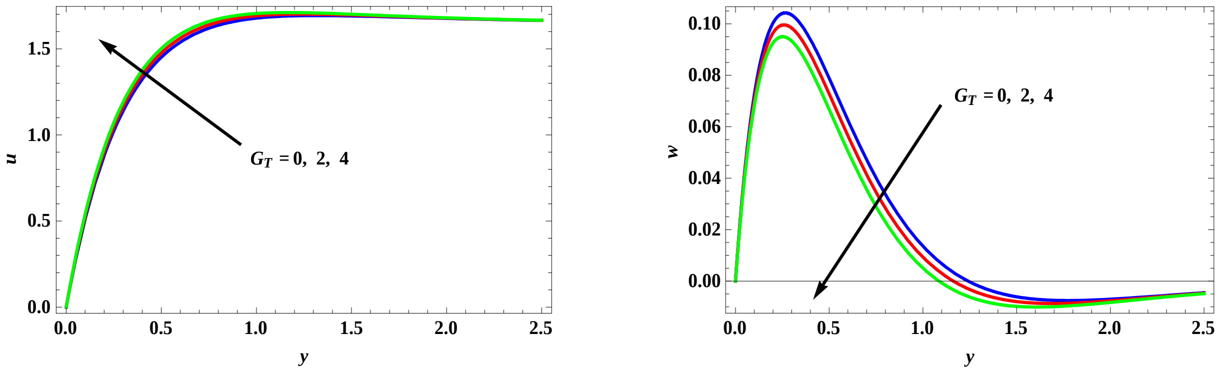
(a) (b)

Fig.4 Velocity profiles in the (a) primary and (b) secondary flow directions when $\beta_e = 0.5, \beta_i = 0.5, E = 1, k_1 = 0.3, G_T = 4, G_C = 5,\ t = 0.5, \Pr = 0.71,\ \phi = 1, Sc = 0.22,\ K_1 = 0.2.$



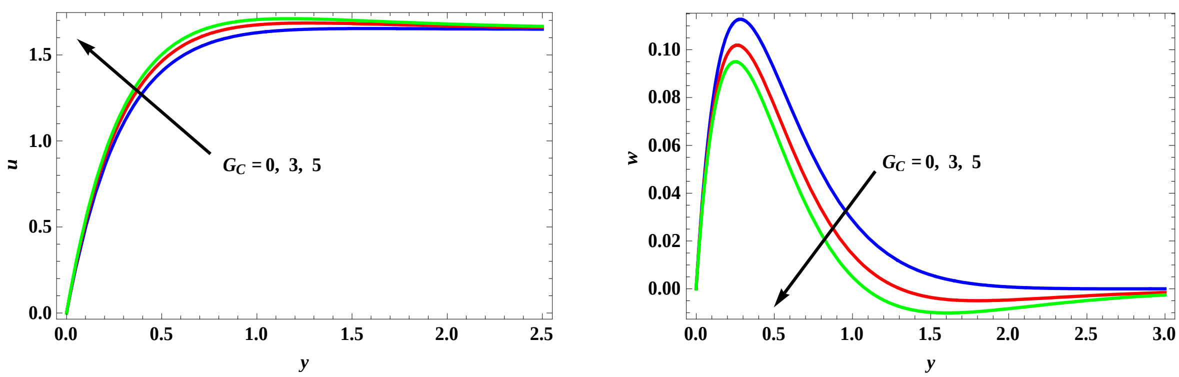
(a) (b)

Fig.5 Velocity profiles in the (a) primary and (b) secondary flow directions when $\beta_e = 0.5, \beta_i = 0.5, E = 1, M = 9, k_1 = 0.3, G_C = 5, t = 0.5, \Pr = 0.71, \phi = 1, Sc = 0.22, K_1 = 0.2.$



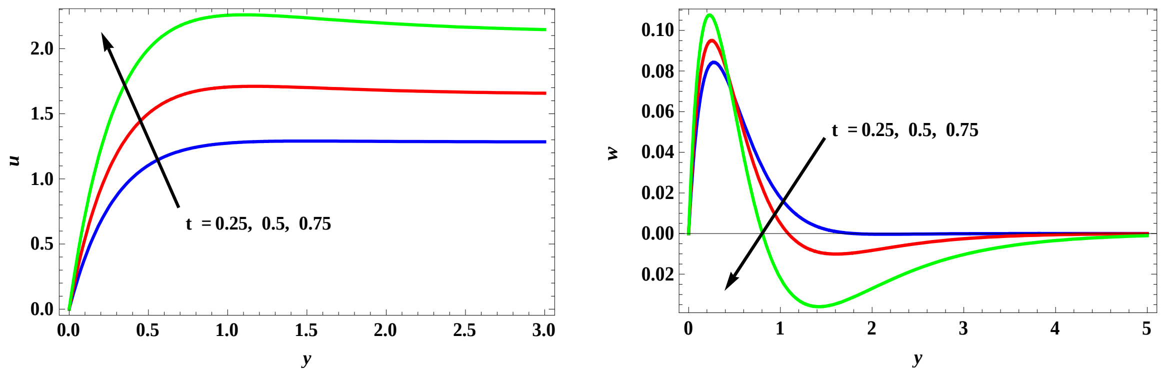
(a) (b)

Fig.6 Velocity profiles in the (a) primary and (b) secondary flow directions when $\beta_e = 0.5, \beta_i = 0.5, E = 1, M = 9, k_1 = 0.3, G_C = 5, t = 0.5, \Pr = 0.71, \phi = 1, Sc = 0.22, K_1 = 0.2.$



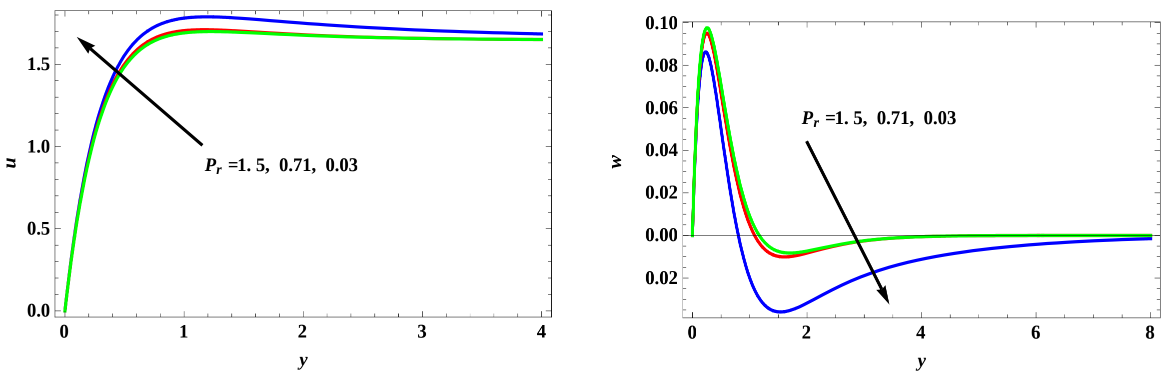
(a) (b)

Fig.7 Velocity profiles in the (a) primary and (b) secondary flow directions when $\beta_e = 0.5, \beta_i = 0.5, E = 1, M = 9, k_1 = 0.3, G_T = 4, t = 0.5, \Pr = 0.71, \phi = 1, Sc = 0.22, K_1 = 0.2.$



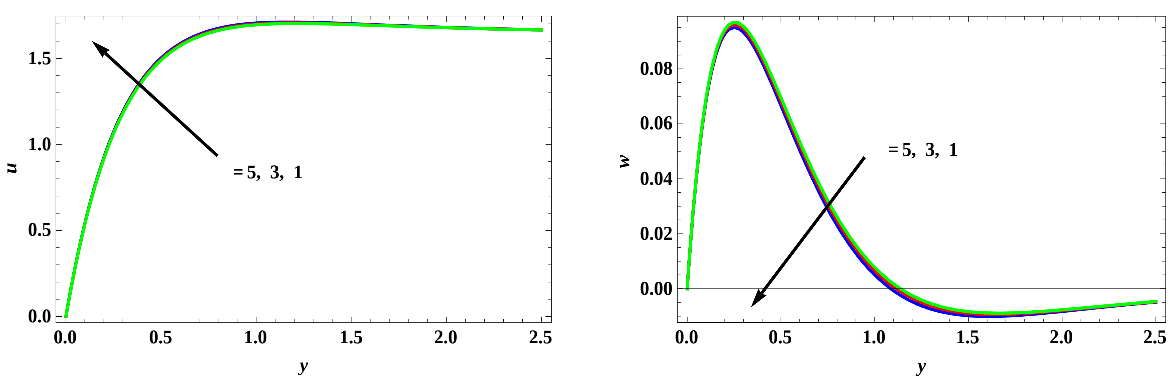
(a) (b)

Fig.8 Velocity profiles in the (a) primary and (b) secondary flow directions when $\beta_e = 0.5, \beta_i = 0.5, E = 1, M = 9, k_1 = 0.3, G_T = 4, G_C = 5, \Pr = 0.71, \phi = 1, Sc = 0.22, K_1 = 0.2.$



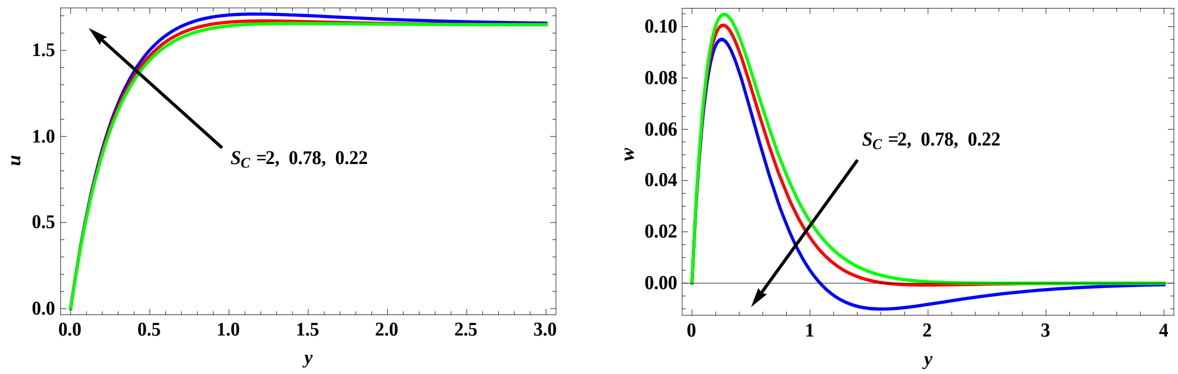
(a) (b)

Fig.9 Velocity profiles in the (a) primary and (b) secondary flow directions when $\beta_e = 0.5, \beta_i = 0.5, E = 1, M = 9, k_1 = 0.3, G_T = 4, G_C = 5, t = 0.5, \phi = 1, Sc = 0.22, K_1 = 0.2.$



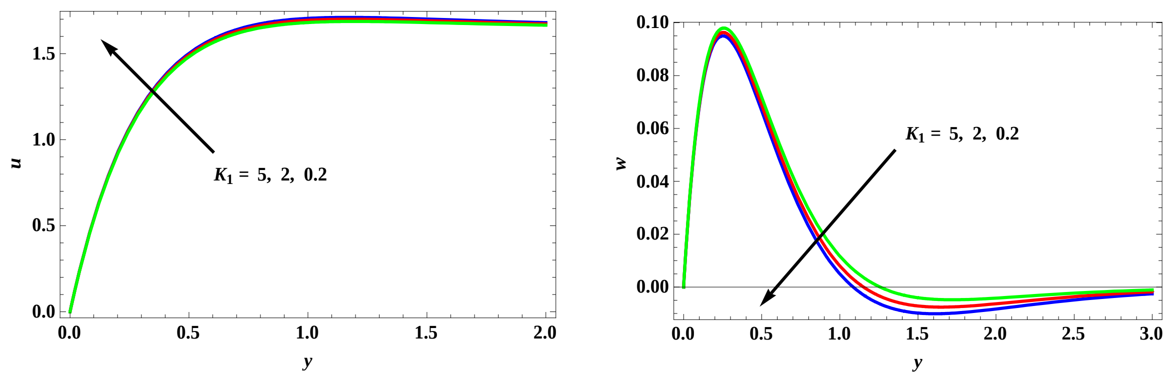
(a) (b)

Fig.10 Velocity profiles in the (a) primary and (b) secondary flow directions when $\beta_e = 0.5, \beta_i = 0.5, E = 1, M = 9, k_1 = 0.3, G_T = 4, G_C = 5, t = 0.5, \Pr = 0.71, Sc = 0.22, K_1 = 0.2.$



(a) (b)

Fig.11 Velocity profiles in the (a) primary and (b) secondary flow directions when $\beta_e = 0.5, \beta_i = 0.5, E = 1, M = 9,\ k_1 = 0.3,\ G_T = 4,\ G_C = 5,\ t = 0.5, \Pr = 0.71,\ \phi = 1,\ K_1 = 0.2.$



(a) (b)

Fig. 12 Velocity profiles in the (a) primary and (b) secondary flow directions when $\beta_e = 0.5, \beta_i = 0.5, E = 1, M = 9,\ k_1 = 0.3,\ G_T = 4,\ G_C = 5,\ t = 0.5, \Pr = 0.71,\ \phi = 1, Sc = 0.22.$

Figs. 13 and 16 respectively, display the variation in the temperature and concentration corresponds to the time. It is observed that temperature and concentration get rise as time passes. Impacts of thermal diffusion on fluid temperature are demonstrated in Fig.14 while the impacts of mass diffusion on concentration are shown in the Fig. 17. Fluid temperature falls down on raising the Prandtl number while concentration falls down on raising the Schmidt number. This concludes that thermal diffusion tends to rise fluid temperature while mass diffusion tends to raise concentration. Also it is noticed that thermal diffusion has tendency to raise thermal boundary layer thickness. Mass diffusion has also similar nature on concentration boundary layer thickness. Figs. 15 and 18, respectively, illustrate the influences of heat absorption on fluid temperature and consequences of chemical reaction on concentration. It can be easily seen that heat absorption has tendency to reduce fluid temperature while chemical reaction has similar tendency on concentration.

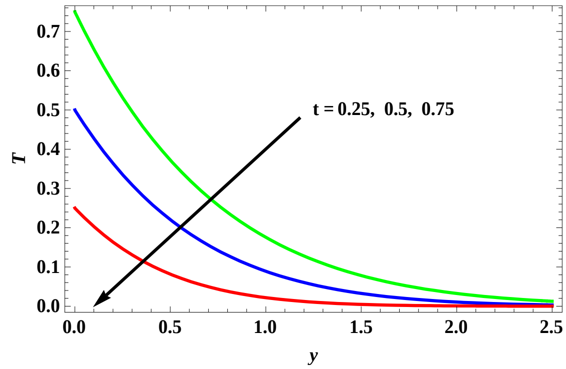


Fig. 13 Temperature profiles when $\Pr = 0.71$ and $\phi = 1$.

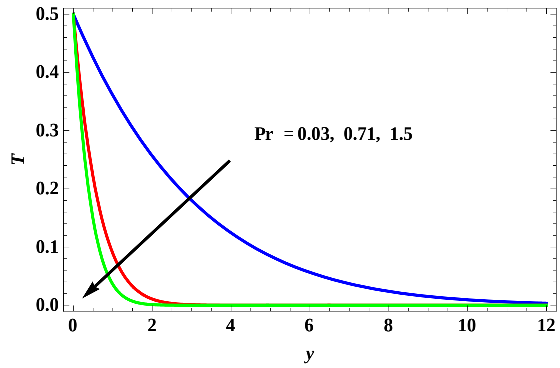


Fig. 14 Temperature profiles when $t = 0.5$ and $\phi = 1$.

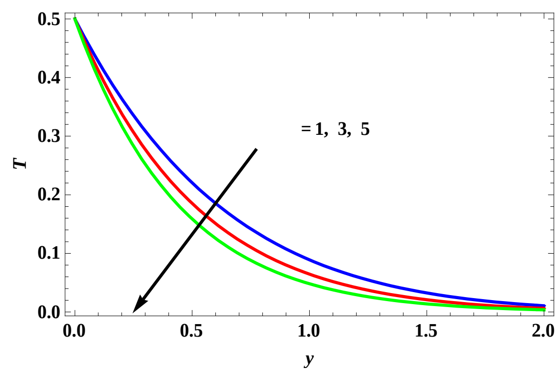


Fig. 15 Temperature profiles when $t = 0.5$ and $\Pr = 0.71$

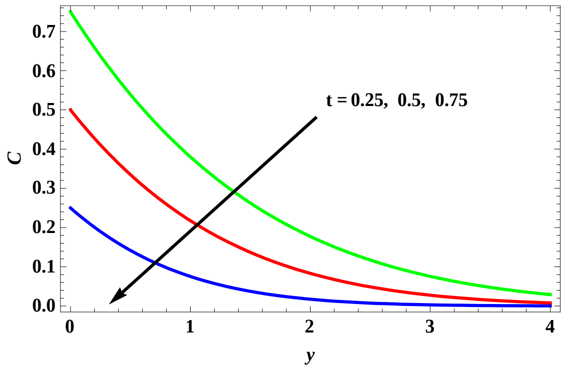


Fig. 16 Concentration profiles when $Sc = 0.22$ and $K_1 = 0.2$.

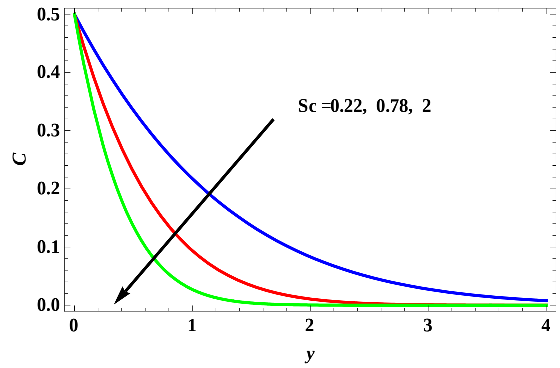


Fig. 17 Concentration profiles when $t = 0.5$ and $K_1 = 0.2$.

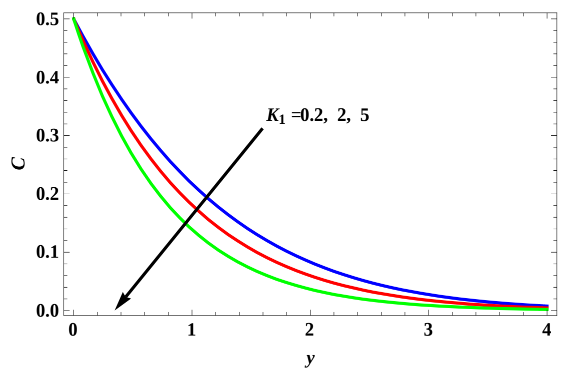


Fig. 18 Concentration profiles when $t = 0.5$ and $Sc = 0.22$.

Table 1 depicts the skin friction coefficient behavior at the plate corresponds to various system parameters. It can be easily seen that Coriolis force, magnetic force, Darcian drag force, thermal and concentration buoyancy forces, time and thermal and mass diffusions have tendency to rise the skin friction coefficient at the plate in the primary flow direction where as Hall and ion-slip currents, heat absorption and chemical reaction tend to reduce it. Skin friction coefficient at the plate in the secondary flow direction rise on raising Hall current, Coriolis force, magnetic force, time, heat absorption and chemical reaction whereas it falls down on raising ion-slip current, Darcian drag force, thermal and concentration buoyancy forces and thermal and mass diffusions. Consequences of various system parameters on the rate of heat and mass transfer behavior at the plate are illustrated in the tables 2 and 3. It is noted that rate of heat and mass transfer get enhanced as time progresses. Rate of heat transfer at the plate fall down on raising the thermal diffusion while rate of mass transfer at the plate fall down on raising mass diffusion. Heat absorption tends to raise rate of heat transfer at the plate while chemical reaction tends to raise rate of mass transfer at the plate.

Table1 Skin friction coefficient at the plate

β_e	β_i	E	M	k_1	G_T	G_C	t	Pr	ϕ	Sc	K_1	$-\tau_x$	τ_z
0.25	0.5	1	9	0.3	4	5	0.5	0.71	1	0.22	0.2	6.6849	0.7802
0.5	**0.5**	**1**	**9**	**0.3**	**4**	**5**	**0.5**	**0.71**	**1**	**0.22**	**0.2**	6.4120	0.9924
0.75	0.5	1	9	0.3	4	5	0.5	0.71	1	0.22	0.2	6.1705	1.0996
0.5	**1**	1	9	0.3	4	5	0.5	0.71	1	0.22	0.2	6.2101	0.8733
0.5	**2**	1	9	0.3	4	5	0.5	0.71	1	0.22	0.2	5.9149	0.7448
0.5	0.5	**2**	9	0.3	4	5	0.5	0.71	1	0.22	0.2	6.5009	1.4108
0.5	0.5	**3**	9	0.3	4	5	0.5	0.71	1	0.22	0.2	6.6106	1.8083
0.5	0.5	1	**12**	0.3	4	5	0.5	0.71	1	0.22	0.2	6.8678	1.0873
0.5	0.5	1	**15**	0.3	4	5	0.5	0.71	1	0.22	0.2	7.2920	1.1760
0.5	0.5	1	9	**0.05**	4	5	0.5	0.71	1	0.22	0.2	9.3187	0.6621
0.5	0.5	1	9	**2**	4	5	0.5	0.71	1	0.22	0.2	5.7750	1.1225
0.5	0.5	1	9	0.3	**0**	5	0.5	0.71	1	0.22	0.2	6.0313	1.0361
0.5	0.5	1	9	0.3	**2**	5	0.5	0.71	1	0.22	0.2	6.2217	1.0142
0.5	0.5	1	9	0.3	4	**0**	0.5	0.71	1	0.22	0.2	5.8477	1.0704
0.5	0.5	1	9	0.3	4	**3**	0.5	0.71	1	0.22	0.2	6.1863	1.0236
0.5	0.5	1	9	0.3	4	5	**0.25**	0.71	1	0.22	0.2	4.6627	0.8185
0.5	0.5	1	9	0.3	4	5	**0.75**	0.71	1	0.22	0.2	8.5324	1.2086
0.5	0.5	1	9	0.3	4	5	0.5	**0.03**	1	0.22	0.2	6.5395	0.9548
0.5	0.5	1	9	0.3	4	5	0.5	**1.5**	1	0.22	0.2	6.3635	1.0033
0.5	0.5	1	9	0.3	4	5	0.5	0.71	**3**	0.22	0.2	6.3895	0.9969
0.5	0.5	1	9	0.3	4	5	0.5	0.71	**5**	0.22	0.2	6.3714	1.0004
0.5	0.5	1	9	0.3	4	5	0.5	0.71	1	**0.78**	0.2	6.3299	1.0149
0.5	0.5	1	9	0.3	4	5	0.5	0.71	1	**2**	0.2	6.2529	1.0321
0.5	0.5	1	9	0.3	4	5	0.5	0.71	1	0.22	**2**	6.3902	0.9977
0.5	0.5	1	9	0.3	4	5	0.5	0.71	1	0.22	**5**	6.3616	1.0046

Table 2 Nusselt number

t	Pr	ϕ	Nu
0.25	0.71	1	0.5140
0.5	**0.71**	**1**	0.7791
0.75	0.71	1	1.0153
0.5	**0.03**	**3**	0.1987

0.5	**1.5**	**5**	1.6416

Table 3 Sherwood number

t	Sc	K_1	Sh
0.25	0.22	0.2	0.2690
0.5	**0.22**	**0.2**	0.3865
0.75	0.22	0.2	0.4809
0.5	**0.78**	**2**	0.9190
0.5	**2**	**5**	0.5994

6. Conclusions

MHD free convection flow of a thermally conducting, chemically reacting and rotating fluid over a vertical plate due to moving free-stream with Hall and ion-slip currents is mathematically discussed. Laplace transform technique is successfully employed to get the solution of resulting PDE's. To analyze the specific flow patterns numerical computation is performed and results are thoroughly discussed with the help of graphs and tables. Some significant out-comes are as follows:

(i) Hall and ion-slip currents have tendency to suppress the primary flow in the neighboring boundary layer region of the plate while this tendency is upturned near the free-stream.

(ii) Thermal and concentration buoyancy forces tend to raise primary flow. Reverse flow induces in the secondary flow direction. On rising the buoyancy forces secondary flow fall down in the boundary layer region adjacent to the plate while this tendency is upturned in the boundary layer region adjacent to the free-stream.

(iii) A remarkable observation recorded that in the absence of thermal buoyancy force there appears reverse flow in the secondary flow direction while in the absence of concentration buoyancy force there does not exist reverse flow in the secondary flow direction.

(iv) Thermal diffusion tends to rise fluid temperature while mass diffusion tends to raise concentration.

(v) Thermal diffusion has tendency to raise thermal boundary layer thickness. Mass diffusion has also similar nature on concentration boundary layer thickness.

The other results discussed in the previous section are also revealing and may find applications in fluid engineering and biomagnetic fluid dynamics.

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